SOURCE LOCALIZATION USING TIME-DIFFERENCE-OF-ARRIVAL WITHIN A SPARSE REPRESENTATION FRAMEWORK

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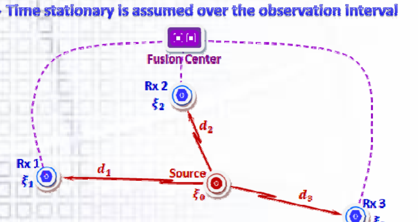
❶ Introduction:

Objective:

- Localization in a plane of a non-cooperative source

Framework:

- Sensors distributed over the plane at arbitrarily, but precisely known locations
- Localization is carried out at a fusion center, assumed linked via ideal communication links to the sensors
- Time synchronization is assumed across sensors
- Signal and timing information of the source are not available at the sensors. The pulse shape is assumed known
- One of the sensors is taken as reference
- The propagation environment is multipath: except for the reference sensor, the line-of-sight (LOS) signal component received at the sensor is obscured by multipath reflections
- Time stationary is assumed over the observation interval



❷ Model and notations:

- Unknown source location: $\xi_0 = [x_0, y_0]^T$
- Known sensor location: $\xi_k = [x_k, y_k]^T$
- LOS propagation delay:

$$\tau_{k0}(\xi_0) = d_k/c = (1/c)\sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}$$

- The time-difference-of-arrival (TDOA) at pairs of sensors:

$$\Delta\tau_{kl}(\xi_0) = \tau_{k0}(\xi_0) - \tau_{l0}(\xi_0)$$

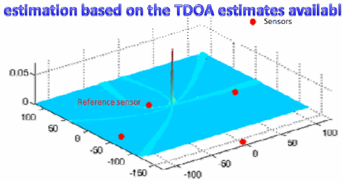
- The signal received at any sensor k is modeled:

$$r_k(t) = (s * h_k)(t) + w_k(t)$$

- The multipath channel: $h_k(t) = \sum_{p=1}^{P_k} \alpha_{kp}\delta(t - \tau_{kp})$

❸ Localization procedure:

- First stage: TDOA estimation (focus of this work)
- Second stage: Hyperbolic localization, i.e., source location estimation based on the TDOA estimates available



❹ TDOA estimation for sparse channels:

Motivation:

- Non-cooperative source ⇨ TDOA estimation is needed
- Multipath propagation ⇨ TDOA estimation is challenging
- Conventional cross-correlation (CC) and sub-space (e.g., root-MUSIC) can solve the problem
- Higher resolution can be obtained by exploiting the sparse structure of the multipath channel (i.e., the number of multipaths is much smaller than the number of samples of the received signal)

ℓ_1-regularization method for TOA:

- Signal model:

$$r_k = Sh_k + w_k \,,$$

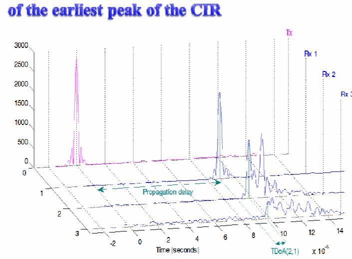
where the received signal vector
$r_k = [r_k(1), \ldots, r_k(Q + L - 1)]^T$,
the CIR vector $h_k = [h_k(1), \ldots, h_k(L)]^T$,
and the noise vector $w_k = [w_k(1), \ldots, w_k(Q + L - 1)]^T$

- S is the $(Q + L - 1) \times L$ matrix relating the received signal vectors r_k to the channel vectors h_k
- $L \gg P_k$ ⇨ the CIR, h_k, is a sparse vector

- TOA estimation as an ℓ_1-regularization problem:

$$\underset{h_k}{\text{minimize}} \;\; \|r_k - Sh_k\|_2^2 + \lambda_k\|h_k\|_1$$

- The time-of-arrival (TOA) can be found as the timing of the earliest peak of the CIR



ℓ_1-regularization method proposed for TDOA:

- Cross-correlation of the signals received at two sensors (dropping the noise term):

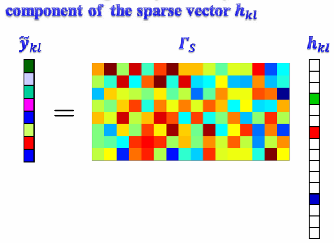
$$\tilde{y}_{kl} = \Gamma_S h_{kl} \,,$$

where $\tilde{y}_{kl} = F y_{kl}$,
F = unitary DFT matrix,
y_{kl} = CC sequence between r_k and r_l ,
$\Gamma_S = \text{diag}\{\tilde{u}_S\}\, F$, of $N \times N$, where $N = 2Q + 2L - 3$,
$\tilde{u}_S$ = the power spectral density of the transmitted signal padded with $L - 1$ zeros

- TDOA estimation as an ℓ_1-regularization problem:

$$\underset{h_{kl}}{\text{minimize}} \;\; \|\tilde{y}_{kl} - \Gamma_S h_{kl}\|_2^2 + \lambda_{kl}\|h_{kl}\|_1$$

- The TDOA is given by the delay of the first component of the sparse vector h_{kl}



Discussion:

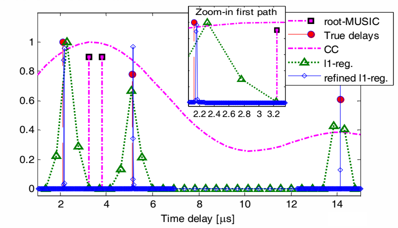
- The ℓ_1-regularization is a convex optimization problem
- The ℓ_1-norm is used as an approximation of the ℓ_0-norm, to impose sparsity on the estimated vector
- The regularization parameter λ balances the fit of the solution to the measurements versus sparsity
- The ℓ_1-regularization may lead to a sub-optimal solution for the overall estimated vector; however, it still produces better TDOA estimation resolution than conventional techniques, especially at low SNR
- An iterative grid refinement approach is adopted to obtain high resolution while keeping the complexity of the algorithm in check
- The proposed estimation method does not require knowledge of the transmitted symbols, nor the number of the multipath components

❺ Simulation results:

Set-up:

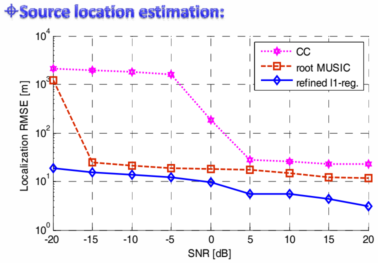
- $M = 8$ sensors, approximately uniformly distributed around the source on a circle of radius 1 km
- The source emits a GMSK signal of bandwidth 200 kHz
- Each sensor sees a multipath channel with three components , except for the reference sensor which receives only the LOS component

TDOA estimation:



- 500 received symbols, sampled 8 times the Nyquist rate are used; the average received SNR is 15 dB per sample
- An oversampling factor of ×50 is used for grid refinement, resulting in the 'refined ℓ_1-reg' curve
- *The estimate by the ℓ_1-regularization method with grid refinement is the closest to the true delays*

Source location estimation:



- The source location is estimated by an semidefinite relaxation method based on the TDOA estimates
- The root-mean-squared-error (RMSE) plotted is obtained from Monte Carlo simulations with 100 runs per SNR value
- *The ℓ_1-regularization method with grid refinement offers better accuracy and is more robust to noise than the conventional methods*

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