Array Processing for Passive Localization of a Source in the Near Field

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***Abstract* — This publication presents a coherent processing method for RF (radio frequency) source localization in a plane by employing a passive sensor array, widely distributed over a given area. The context of the localization technique discussed involves placement of the source in the near field of the sensor array and unknown character of the source in the sense that the transmitted signal, its power spectral density, and its timing are unknown to the sensors. However, the sensors have ideal mutual time and phase synchronization, highly precise knowledge of their own location, and ideal communication link to a fusion center. The source location is centrally estimated by coherently processing the received signals. For this, an estimator is derived exploiting the source location information in the phase difference of the received signals at pairs of sensors. An analytical expression is developed for the Cramer Rao lower bound (CRLB) relating the mean square error (MSE) of the location estimates to the source signal parameters and sensors layout. Numerical examples and Monte Carlo analysis validate the close form expression developed and show the high accuracy capabilities of the source localization via the presented coherent processing method.**

I. INTRODUCTION

Source localization using sensor arrays has been a problem of high interest in various fields, such as radar, sonar, navigation, acoustics, geophysics, or other sensor networks for the past few decades. Due to technology advances and new requirements in terms of accuracy and channels over which the propagation takes place, the localization problem remains a highly active research topic. The range of possible applications, as well as that of the localization techniques, is very wide. Both are clearly presented in many overviews in the literature, e.g., [1]. Of interest in the current publication is the passive localization in plane of unknown sources, i.e., sources for which the actual signal, its power spectral density, and time and phase of the transmitted signal are unknown to the sensors. Extensive work has been carried for estimation of the direction of arrival (DoA) of sources placed in the far field of a sensor array. For sources placed in the *near field* of a sensor array, i.e., the sensors are widely dispersed over the source surrounding area, both the bearing and the range can be estimated for source localization. Conventional localization methods generally exploit amplitude or time delay information contained in the envelope of the received signals. Received signal strength (RSS), time of arrival (ToA), and time difference of arrival (TDoA) based are among the well known localization techniques [1]. Since these exploit only the envelope of the received signals, we refer to them collectively as *non-coherent*. In this paper we study a *coherent* approach to localization by additionally exploiting the relative carrier phases of the received signals among pairs of sensors. The localization is accomplished by formulating a *localization metric* which is a joint statistic that incorporates the phase information contained in the received signals as if transmitted from various points of the source two-dimensional space. The source space can be limited by a previously non-coherent source location estimation step.

The rest of the paper is organized as follows: the system model considered is presented in Section II. The higher source localization accuracy capabilities of the coherent processing over the non-coherent one is shown in Section III. Section IV is dedicated to the performance analysis through the CRLB expression, backed-up by Monte Carlo simulations in Section V. Section VI contains the formulated conclusions.

II. SYSTEM MODEL

With passive localization the unknown x-y location ξ_0 of an emitting source has to be estimated based on the signals collected by a number M of sensors. The source is assumed to transmit an unknown lowpass signal $s_0(t)$ modulating a carrier frequency f_c. The signal is assumed narrow-band in the sense that the carrier frequency is much higher than the signal's bandwidth. The sensors are widely dispersed within a surveillance area, at precisely known arbitrarily fixed locations ξ_k, forming a distributed sensor array. The source is in the near-field of the distributed array in the sense that it has a different bearing, and possibly a different range, with respect to each of the sensors. Ideal mutual time and phase synchronization are assumed across the sensors. These allow complete source localization by coherent processing, i.e., by processing both the envelope and the carrier phase measurements at the sensors. All processing is carried out at a fusion center assumed linked via ideal communication links to the sensors. Both the envelope and the carrier phase measurements are related to the source location by the embedded time delay. The time delay between the source at ξ_0 and a sensor at ξ_k is given by

$$\tau_k(\xi_0) = \frac{1}{c} d_k(\xi_0) = \frac{1}{c}\sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}\ , \qquad (1)$$

where c is the speed of light and $d_k(\xi_0)$ is the travelled distance between the two locations.

The model for the signal received at a sensor is expressed

$$r_k(t) = \alpha_k\, s_0\big(t - \tau_k(\xi_0)\big) + w_k(t), \tag{2}$$

where α_k is the complex-valued channel gain (pathloss due to source-sensor separation plus carrier phase shift) and $w_k(t)$ is additive white Gaussian noise (AWGN), with variance σ^2, $w_k(t) \sim \mathcal{N}(0, \sigma^2)$. The system is assumed stationary over the observation time interval such that α_k and τ_k are time invariant over the aforementioned interval. The complex gain is expressed

$$\alpha_k = g_k\; e^{-j\omega_c \tau_k(\xi_0)}, \tag{3}$$

where $\omega_c = 2\pi f_c$ and g_k is the real-valued gain (in fact attenuation) of the transmitted signal through the propagation channel from the source to a sensor. Given that the signal arrives at a sensor through the line-of-sight (LOS) path from the source, it is reasonable to assume g_k as being dependent only on the free space propagation path loss, which varies with the source location. The carrier phase term $-\omega_c \tau_k(\xi_0)$ is a demodulation residue and it depends on the carrier frequency and the unknown propagation delay, which is also dependent on the source location according to (1). The variation of g_k with the source location is observed to be much slower than that of the phase term. Furthermore, g_k can be estimated, for example, by direct measurement of the received power, and employing a path-loss model. As such, g_k is assumed known.

For localization metric derivation, it is useful to model the system in the frequency domain. The received signal at the k^{th} sensor, can be written in frequency by applying the Fourier transform on (2):

$$R_k(\omega) = \Gamma_k(\omega) S_0(\omega) + W_k(\omega), \tag{4}$$

where $\Gamma_k(\omega) = g_k\, e^{-j\omega\tau_k(\xi_0)} e^{-j\omega_c\tau_k(\xi_0)}$ and $W_k(\omega)$ is the frequency domain correspondent of the AWGN noise. Putting together the spatial samples from all the M sensors into one frequency domain snapshot, a vectorial form of (4) can be written for each frequency bin of interest ω, as

$$\boldsymbol{R}(\omega) = \boldsymbol{\Gamma}(\omega) S_0(\omega) + \boldsymbol{W}(\omega), \tag{5}$$

with $\boldsymbol{R}(\omega) = \left[R_1(\omega) \ldots R_M(\omega)\right]^T$, $\boldsymbol{\Gamma}(\omega) = \left[\Gamma_1(\omega) \ldots \Gamma_M(\omega)\right]^T$, and $\boldsymbol{W}(\omega) = \left[W_1(\omega), \ldots, W_M(\omega)\right]^T$.

III. Coherent Location Estimation

Coherent processing of the collected signals for source location direct estimation involves a two-dimensional search for the maximum of a localization metric among all the possible plane locations of the source. Such a metric can be obtained based on the maximum likelihood (ML) procedure. Due to the phase term in the received signal model (4), the derivation can be more conveniently carried in frequency domain. Joint probability density function (pdf) of the noise across the sensors, at frequency ω

$$f_\omega\big(\boldsymbol{W}(\omega)\big) = \frac{1}{\pi^M \det\big(\boldsymbol{K}_W(\omega)\big)} \exp\left\{-\boldsymbol{W}^H(\omega) \boldsymbol{K}_W^{-1}(\omega) \boldsymbol{W}(\omega)\right\}, \tag{6}$$

where $\boldsymbol{K}_W(\omega)$ is the covariance matrix of the noise across the sensors, defined as $\mathbb{E}\left\{\boldsymbol{W}(\omega)\boldsymbol{W}^H(\omega)\right\} = \sigma^2 \boldsymbol{I}_M$, $\boldsymbol{I}_M$ being the M-dimensional identity matrix. The signal transmitted by source is deterministic unknown, i.e., no statistical model is assumed for it. The signal model (5) is used in (6) to express the joint pdf of the received signals across sensors, for all frequencies ω within the set B_0 of interest, given any source location ξ and source signal $S_0(\omega)$:

$$f\left(\boldsymbol{R} \middle| \xi, \boldsymbol{S}_0\right) = \prod_{\omega \in B_0} f_\omega\left(\boldsymbol{R}(\omega) \middle| \xi, S_0(\omega)\right), \tag{7}$$

where it was considered that the received signal has independent distributions over frequencies of interest.

The ML estimation of the source location is given by the following optimization criterion:

$$\hat{\xi}_0 = \arg\max_{\xi} \Lambda(\xi). \tag{8}$$

After substituting $S_0(\omega)$ with an estimate and dropping location independent terms, similar to the derivation carried in [2], the source location can be estimated by maximizing the log-likelihood

$$\begin{aligned} \Lambda(\xi) &= \sum_{\omega \in B_0} \boldsymbol{R}^H(\omega) \boldsymbol{\Gamma}(\omega) \boldsymbol{\Gamma}^H(\omega) \boldsymbol{R}(\omega)\, d\omega = \\ &= \sum_{\omega \in B_0} \left| \sum_{l=1}^{M} \alpha_l^* R_l(\omega) e^{j\omega\tau_l(\xi)} \right|^2 d\omega. \end{aligned} \tag{9}$$

After expanding (9) and again keeping only the terms dependent on the source location, the log-likelihood to be maximized is

$$\Lambda(\xi) = \sum_{k=1}^{M} \sum_{l=k+1}^{M} \sum_{\omega \in B_0} \operatorname{Re}\left\{\alpha_l^* \alpha_k\; R_l(\omega) R_k^*(\omega) e^{-j\omega\Delta\tau_{kl}(\xi)}\right\} d\omega, \tag{10}$$

where $\operatorname{Re}\{\cdot\}$ denotes the real part. One may note that $R_l^*(\omega) R_k(\omega)$ represents the discrete Fourier transform of the cross-correlation of signals $r_l(t)$ and $r_k(t)$, denoted $x_{lk}(\tau)$.

With this, a localization metric for coherent processing is formulated in time domain as

$$\Lambda(\xi)=\sum_{k=1}^{M}\sum_{l=k+1}^{M}\mathrm{Re}\left\{g_l\,g_k\,x_{lk}^{*}\left(\Delta\tau_{kl}(\xi)\right)e^{-j\omega_c\Delta\tau_{kl}(\xi)}\right\}. \quad (11)$$

This accounts for both the envelope and carrier phase of the collected signals. A non-coherent system instead is able to process only the received envelopes. As such, for a non-coherent system the channel gain is real-valued, i.e., $\alpha_k=g_k$. Consequently, a non-coherent localization metric is expressed

$$\Lambda^{(nC)}(\xi)=\sum_{k=1}^{M}\sum_{l=k+1}^{M}\mathrm{Re}\left\{g_l\,g_k\,x_{lk}^{*}\left(\Delta\tau_{kl}(\xi)\right)\right\}. \quad (12)$$

The coherent localization metric (11) is similar to the non-coherent localization metric (12), except for the term $e^{-j\omega_c\Delta\tau_{kl}(\xi)}$, which aims to compensate for the carrier phase difference at each pair of sensors. This term sets the premise to the high accuracy localization capabilities of the coherent processing over the non-coherent. This is also graphically illustrated in Fig. 1 for a ratio $\omega_c/B_0\approx 5000$ and a received $SNR=10\text{ dB}$, where the coherent and non-coherent localization metrics were plotted for $M=16$ sensors uniformly placed on a virtual circle of radius 500 m around the source. By comparing the two plots it becomes evident how coherent processing enhances the localization accuracy by substantially narrowing the mainlobe of the localization metric.

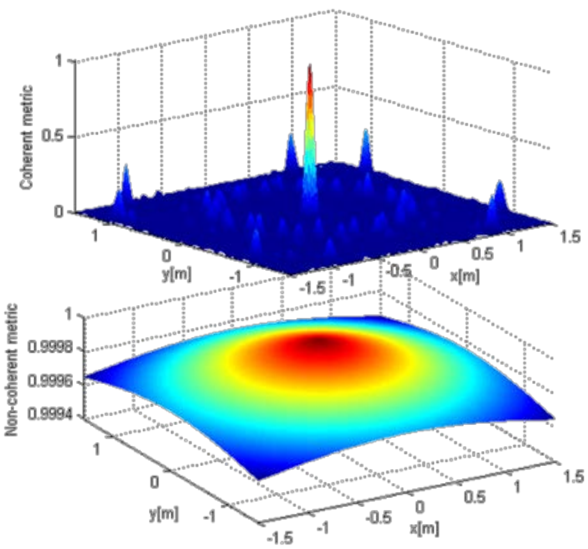

Fig. 1. Coherent processing resolution capabilities improvement over non-coherent processing

I. Cramer Rao Lower Bound

The most common measure of the performance of a localization algorithm is the square root of the MSE (root-MSE). Since to obtain a MSE figure of merit in noise extended computer simulations are required, finding a lower bound for it is extremely useful. It can be shown that the MSE of the source location estimation is lower bounded by

$$\mathbb{E}_{\boldsymbol{R}}\left\{\left\|\hat{\xi}_0-\xi_0\right\|^2\right\}=\mathrm{var}\left\{\hat{\xi}_0\right\}=\mathrm{var}\left\{x_0\right\}+\mathrm{var}\left\{y_0\right\}\geq\eta_R\rho_L, \quad (13)$$

where $\rho_L=(q_x+q_y)/(q_xq_y-p_{xy}^2)$, given that

$$\boldsymbol{C}_{CRLB}(\xi_0)=\frac{\eta_R}{q_xq_y-p_{xy}^2}\begin{bmatrix}q_x & p_{xy}\\ p_{xy} & q_y\end{bmatrix}, \quad (14)$$

with
$$q_x=\sum_{k=1}^{M}SNR_k\sin^2\varphi_k-\frac{1}{\sum_{k'=1}^{M}SNR_{k'}}\left(\sum_{k=1}^{M}SNR_k\sin\varphi_k\right)^2, \quad (15)$$

$$q_y=\sum_{k=1}^{M}SNR_k\cos^2\varphi_k-\frac{1}{\sum_{k'=1}^{M}SNR_{k'}}\left(\sum_{k=1}^{M}SNR_k\cos\varphi_k\right)^2, \quad (16)$$

$$p_{xy}=-\sum_{k=1}^{M}SNR_k\sin\varphi_k\cos\varphi_k+\\+\frac{1}{\sum_{k'=1}^{M}SNR_{k'}}\left(\sum_{k=1}^{M}SNR_k\sin\varphi_k\right)\left(\sum_{k=1}^{M}SNR_k\cos\varphi_k\right). \quad (17)$$

The lower bound on the source localization MSE given by (13) contains essentially two factors. The first one, η_R, shows the effect of the signal bandwidth and carrier frequency, which is similar to the case of high resolution MIMO radar [5]. For narrow-band signals, i.e., $\beta_{\omega_c}\approx 1$, the effect of the signal bandwidth is negligible. Instead, the inverse proportionality with ω_c leads to the conclusion that the coherent processing offers much higher accuracy capabilities than the non-coherent processing. However, this conclusion is based on the CRLB, which is known as being a tight bound at high SNR only and being a bound of small errors [5]. As such, it ignores effects that could lead to large errors, like the high sidelobes, characteristic to the coherent processing [6], [7].

The second factor in (13), ρ_L, shows the effect of the geometric relations between the source and the sensors, impacted by the number of sensors and the SNR at these sensors.

II. Numerical Examples

While the best achievable performance of the estimation is indicated by the CRLB, the MSE of the ML estimate is close to the CRLB only at high SNR [13]. A threshold effect was observed in location estimation systems, meaning essentially that there is a threshold value of the SNR, above which is the asymptotic region, where the estimation errors are small and the MSE is close to the CRLB [8], [9]. Otherwise, in the non-asymptotic region, the MSE rises quickly and deviates significantly from the CRLB. This behavior can be observed in Fig. 2, where a system of $M = 8$, respectively 16 sensors has been employed to localize a GSM source (the transmitted signal is GMSK and has a bandwidth of $200\ \text{kHz}$, while the carrier frequency is $f_c = 980\ \text{MHz}$) situated within a known area of 50 m by 50 m. The sensors have been uniformly placed on a virtual circle with radius of 500 m around the source location. For each of the SNR values considered 100 simulations were performed. The threshold effect can be observed. Below the threshold, the root-MSE increases up to some value close to the limitation imposed by the a priori known area within which the source is placed. The root-MSE increasing as the SNR decreases below the threshold means that the large localization errors take the place of the small ones. As expected, above the threshold, the root-MSE follows closely the CRLB and one may note that these values are below 1 m, while for the non-coherent systems the best achievable performance is tens of meters [17].

While from the close form expression of the CRLB (13) it may not be evident, by numerical evaluation, it can be shown that increasing the number of sensors improves the performance of the system. In Fig. 2 it is shown that an increase from 8 to 16 in the number of sensors, can bring, according to the CRLB, a performance improvement of about 3 dB in terms of MSE. The MSE also shows that the SNR threshold moved from -2 dB to -15 dB by increasing the number of sensors from 8 to 16.

III. Conclusions

This paper discusses a coherent processing technique for the planar localization of an unknown radio source in the near field of a widely distributed passive sensor array. A coherent localization metric for deterministic unknown source signal is proposed. The CRLB for the MSE is also derived. The expression obtained is consistent with the results presented in literature for non-coherent processing using passive sensor arrays and for coherent processing using active arrays. As such, the accuracy of the localization is strongly dependent on the carrier frequency and the sensor layout. The numerical examples of CRLB are in accordance with the computer simulations for the root-MSE. At low SNR, the performance is dominated by noise, with false peaks popping up in the localization metric anywhere in the a priori parameter space of the source location. At high SNR the performance is ambiguity free and the CRLB tightly bounds the MSE. Increasing the number of sensors increase the accuracy at high SNR and also expands the ambiguity free region.

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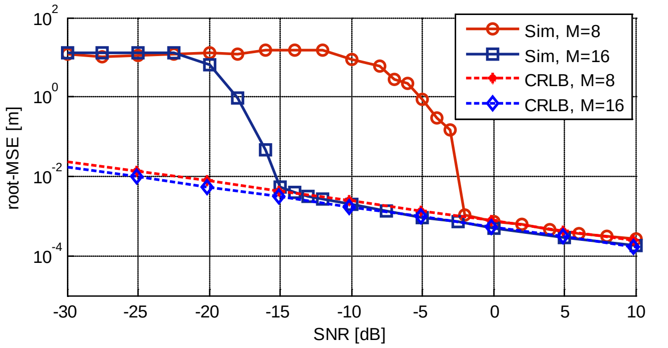

Fig. 2. Localization accuracy for an array of 8 and 16 sensors, respectively. Sensors are uniformly placed on a virtual circle around the source

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