SOURCE LOCALIZATION USING TIME DIFFERENCE OF ARRIVAL WITHIN A SPARSE REPRESENTATION FRAMEWORK

*Ciprian R. Comsa**, *Alexander M. Haimovich**, *Stuart Schwartz*†, *York Dobyns*†, *and Jason A. Dabin*‡

*CWCSPR Lab, ECE Dept, New Jersey Institute of Technology, Newark, NJ, USA
†EE Dept, Princeton University, Princeton, NJ, USA
‡U.S. Army Communications-Electronics Research, Development and Engineering Center Fort Monmouth, NJ, USA
email: ccomsa@ieee.org, web: www.ccspr.njit.edu

ABSTRACT

The problem addressed is source localization via time-difference-of-arrival estimation in a multipath channel. Solving this localization problem typically implies cross-correlating the noisy signals received at pairs of sensors deployed within reception range of the source. Correlation-based localization is severely degraded by the presence of multipath. The proposed method exploits the sparsity of the multipath channel for estimation of the line-of-sight component. The time-delay estimation problem is formulated as an $\ell 1$-regularization problem, where the $\ell 1$-norm is used as a channel sparsity constraint. The proposed method requires knowledge of the pulse shape of the transmitted signal, but it is blind in the sense that information on the specific transmitted symbols is not required at the sensors. Simulation results show that the proposed method delivers higher accuracy and robustness to noise compared to conventional or even super-resolution MUSIC time-difference-of-arrival source localization methods.

Index Terms — Source localization, time-difference-of-arrival, sparse multipath channel, $\ell 1$-regularization

1. INTRODUCTION

Accurate localization of a signal source is a problem of interest in various applications, [1]. The current work addresses the problem of localizing a signal source by sensors distributed over an area. Typically, the source location is estimated in two stages. During the first stage, a measure of the received signal, usually the propagation time-delay, is estimated at each sensor. In the second stage, the actual location is computed from the time delay estimates. Time-delay-estimation (TDE) becomes challenging in multipath propagation environments, where the line-of-sight (LOS) signal component becomes obscured by multipath reflections. Hence, accurate localization requires techniques capable of resolving the LOS signal component. When the transmitted signal and its transmission time are known at a sensor, the time of arrival (TOA) can be estimated by a variety of techniques. A classical method is to estimate the TOA from the timing of the peak of the cross-correlation (CC) between the transmitted and received signals, [2]. The resolution of the TOA estimated in this case is limited by the width of the main lobe of the time autocorrelation function of the transmitted signal. This limitation makes the method unable to distinguish between the LOS signal and a reflected component when they are spaced closer than the resolution limit. Over the years, various techniques have been proposed to overcome this limitation. An example is the root-MUSIC method, belonging to the larger class of subspace methods, also referred to as super-resolution methods due to their high resolution capabilities, [3].

Recently, some potentially even higher resolution estimation techniques have been proposed, based on the observation that many propagation channels associated with multipath environments tend to exhibit a sparse structure in the time domain, i.e., the number of multipaths is much smaller than the number of samples of the received signal. This sparsity has been exploited in TOA estimation, [4], and other TOA-related applications, such as compressed channel sensing, [5, 6], or underwater acoustic channel deconvolution, [7]. TOA estimation requires the transmitted signal to be known to the sensors. In many applications, the source may be non-cooperative or otherwise the signal and timing information may not be available at the receiving sensors. The common approach for such a case is to take one of the sensors as reference and measure the time-difference-of-arrival (TDOA) at each of the other sensors with respect to the chosen reference sensor. A method for TDOA estimation for sparse non-negative acoustic channels is presented in [8].

In this paper, a method for high resolution TDOA estimation for complex-valued sparse multipath channels is developed and applied to source localization. The proposed method casts the TDOA estimation as a convex optimization problem that can be efficiently solved by conventional algorithms, [9]. In particular, the problem is formulated as an $\ell 1$-regularization problem, i.e., the $\ell 1$-norm is used to impose a sparsity-constraint on the channel. While the proposed approach does not require the transmitted signal to be known at the sensors side, as is the case in [4-7], the pulse shape is assumed known. Also, for simplicity, the reference sensor is considered single-path, i.e., the reference sensor receives only the LOS signal. In [10], TOA estimation is carried out also based on knowledge of only the pulse shape. Unlike our approach, where sparsity is exploited to improve localization performance, in [10] the sparsity of the channel is applied to reduce the sampling rate of the received analog signal. The actual time delay estimation is performed by a subspace method.

The remainder of this paper is organized as follows. Sec. 2 introduces the signal model. In Sec. 3, the $\ell 1$-regularization method for TDOA estimation is proposed and discussed. In Sec. 4, numerical simulations are conducted to compare the performance of our method with other techniques for source localization. Conclusions are listed in Sec. 5.

2. SIGNAL MODEL

With the localization problem, the unknown x-y location, ξ_0, of a signal source has to be estimated based on the signals collected by a number M of sensors. The source is assumed to transmit an unknown lowpass signal, $s(t)$, of bandwidth B. The receiving sensors are widely dispersed within a surveillance area, at arbitrary but precisely known locations, ξ_k. The signal received at any sensor is expressed as the convolution between the transmitted signal, $s(t)$, and the channel impulse response (CIR), $h_k(t)$:

$$r_k(t) = (s * h_k)(t) + w_k(t), \tag{1}$$

where $w_k(t)$ is additive white Gaussian noise (AWGN), with variance σ_k^2. The multipath channel is modeled

$$h_k(t) = \sum_{p=1}^{P_k} \alpha_{kp}\delta(t - \tau_{kp}), \tag{2}$$

where $\delta(\cdot)$ denotes the delta function, P_k is the number of paths of the channel observed at sensor k, and α_{kp} is the complex valued channel gain. The channel parameters P_k and α_{kp} are unknown to the sensors.

The localization method proposed here is based on the estimation of the TDOA at pairs of sensors, $\Delta\tau_{kl}(\xi_0) = \tau_{k0}(\xi_0) - \tau_{l0}(\xi_0)$. The LOS propagation delay between the source located at ξ_0 and any sensor at ξ_k, $\tau_{k0}(\xi_0)$, is proportional to the source-to-sensor distance: $\tau_{k0}(\xi_0) = (1/c)\sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}$, where c is the speed of light. A TDOA measurement localizes the source on a hyperboloid with a constant range difference between the two sensors, k and l. Since the source can occupy only one point on the hyperbolic curve, TDOA measurements from the other sensors are used to resolve the location ambiguity. Processing is carried out at a fusion center, assumed linked via ideal communication links to the sensors. Ideal time synchronization is assumed between the sensors. One of the sensors, say $l = 1$, is chosen as reference such that the sensor pairs used for TDOA estimation are $\{k, 1\}$, for $k = 2, \dots, M$.

3. TDOA ESTIMATION FOR SPARSE CHANNELS

The focus of this paper is on improving the TDOA resolution and accuracy and, implicitly, the source localization resolution and accuracy. Recent work has shown that channel estimation can be improved through sparsity regularization, [4-7]. In this section, we propose an $\ell 1$-regularization method for TDOA estimation, exploiting the sparsity of multipath channels.

3.1. ℓ1-regularization method

Assuming for simplicity of presentation that the time-delays of the CIR are integer multiples of the sampling rate, define the received signal vector $\boldsymbol{r}_k = [r_k(1), \dots, r_k(Q + L - 1)]^T$, the CIR vector $\boldsymbol{h}_k = [h_k(1), \dots, h_k(L)]^T$, and the noise vector $\boldsymbol{w}_k = [w_k(1), \dots, w_k(Q + L - 1)]^T$, where $Q + L - 1$, L, and Q are the lengths of the received signal vector, channel and transmitted signal vector, respectively. With these definitions, the signal model (1) can be written

$$\boldsymbol{r}_k = \boldsymbol{S}\boldsymbol{h}_k + \boldsymbol{w}_k, \tag{3}$$

where $\boldsymbol{S}$ is the $(Q + L - 1) \times L$ matrix relating the received signal vectors $\boldsymbol{r}_k$ to the channel vectors $\boldsymbol{h}_k$. Since typically $L \gg P_k$, the CIR, $\boldsymbol{h}_k$, is a sparse vector. Sparsity of the CIR vector can be enforced by minimizing its $\ell 0$-norm, i.e., the number of non-zero elements. Minimization of the $\ell 0$-norm of $\boldsymbol{h}_k$ is a non-convex optimization problem and it is NP-hard, which means that no known algorithm for solving this problem is significantly more efficient than an exhaustive search over all subsets of entries of $\boldsymbol{h}_k$. In lieu of the $\ell 0$-norm, an approximation, e.g., the ℓq-norm, can be used with $0 < q \leq 1$. While smaller q implies better approximation of the $\ell 0$-norm, $q = 1$ is often used because minimization of the $\ell 1$-norm is a convex problem, and it can be efficiently solved by standard algorithms. Thus, assuming that the transmitted signal, and hence the matrix $\boldsymbol{S}$, are known, the CIR estimation can be formulated as an $\ell 1$-regularization problem [7],

$$\min_{\boldsymbol{h}_k} \ \|\boldsymbol{r}_k - \boldsymbol{S}\boldsymbol{h}_k\|_2^2 + \lambda_k \|\boldsymbol{h}_k\|_1, \tag{4}$$

where $\|\boldsymbol{v}\|_q = \sqrt[q]{\sum_i |v_i|^q}$ denotes the ℓq-norm of vector $\boldsymbol{v}$.

The estimate of the CIR can be used to find the TOA as the timing of the earliest peak of the CIR. We now seek to formulate the problem of TDOA estimation. The TDOA has to be determined from a sufficient statistic involving signals received at two sensors. A common such statistic is cross-correlation of the received signals, implying that the TDOA has to be estimated from the cross-correlation of the CIR of two channels, e.g., from $h_{kl}(t)$. For a single channel, the TOA is determined as the time of the first path of the estimated channel. However, when cross-correlating two CIR's, the time of the first path in the cross-correlation does not necessarily correspond to the TDOA. Assuming that for each of the channels, the line-of-sight path is the strongest, the TDOA can be found from the time of the strongest component of the cross-correlation. Here, to simplify the situation, we assume that one of the sensors does not experience multipath, and use this sensor as reference for TDOA estimation. In this case, the TDOA is given by the delay of the first time component of $h_{kl}(t)$. Cross correlating the signal received at sensor k with the reference sensor l, and dropping the noise term for simplicity, we have:

$$\tilde{\boldsymbol{y}}_{kl} = \boldsymbol{\Gamma}_S \boldsymbol{h}_{kl}, \tag{5}$$

where $\tilde{\boldsymbol{y}}_{kl} = \boldsymbol{F}\boldsymbol{y}_{kl}$, $\boldsymbol{F}$ is the unitary discrete Fourier transform (DFT) matrix, and $\boldsymbol{y}_{kl}$ is the cross-correlation sequence of the received signal vectors $\boldsymbol{r}_k$ and $\boldsymbol{r}_l$. Let $N = 2Q + 2L - 3$ be the number of elements of the vector $\boldsymbol{y}_{kl}$. The matrix $\boldsymbol{\Gamma}_S$ is a $N \times N$ transformation matrix relating the frequency domain cross-correlations of the received signals, $\tilde{\boldsymbol{y}}_{kl}$, to the time-domain cross-correlations of the channels, $\boldsymbol{h}_{kl}$. It can be verified that $\boldsymbol{\Gamma}_S = \mathrm{diag}\{\tilde{\boldsymbol{u}}_S\}\,\boldsymbol{F}$, where $\tilde{\boldsymbol{u}}_S$ is the power spectral density of the transmitted signal padded with $L - 1$ zeros.

The problem of TDOA estimation can be formulated as an $\ell 1$-regularization problem:

$$\min_{\boldsymbol{h}_{kl}} \ \|\tilde{\boldsymbol{y}}_{kl} - \boldsymbol{\Gamma}_S \boldsymbol{h}_{kl}\|_2^2 + \lambda_{kl} \|\boldsymbol{h}_{kl}\|_1, \tag{6}$$

which may be efficiently solved with conventional convex optimization algorithms, [9]. Note the presence of the auto-correlation of the transmitted signal within the cost function. The proposed TDOA method utilizes auto-correlation information (for uncorrelated symbols, pulse shape information is sufficient), but the method is blind in the sense that it does not require knowledge of the transmitted symbols.

Formulating (3) with a denser sampled channel has the potential of a higher resolution TDOA estimate, but increases the complexity of the optimization algorithms. An iterative grid refinement approach is adopted to keep the complexity of the optimization algorithms in check. Initially, (6) is solved for the samples corresponding to a desired range of delays. A refined grid is obtained by taking a second set of samples focusing on a range of delays that are indicated by the first iteration to contain multipath. This corresponds to a higher sampling rate of the smaller area of interest. Samples of the second set are obtained by interpolating the original samples. The transformation matrix $\boldsymbol{\Gamma}_S$ is also recalculated to match the refined sample support of the correlation sequences. With the refined grid, (6) is solved again and a new TDOA estimate, of higher resolution, is obtained. The grid refinement procedure can be repeated until a desired resolution is attained.

3.2. Discussion

The cost function to be minimized in $\ell 1$-regularization problems, e.g., (4) and (6), has two terms: the first term is a *measurement fidelity* (or reconstruction error); the second term is a *regularization* (or penalization) *term*, that imposes sparsity on the estimate by using its $\ell 1$-norm. The factor λ is a *regularization parameter*. The sparsity of the solution is governed by the choice of λ, which balances the fit of the solution to the measurements versus sparsity, [11]. A small regularization parameter corresponds to a good fit to the measurements, while too much regularization (over-penalization through a large λ) produces sparser results, but may fail to explain the measurements well. A number of methods have been studied in the literature for automated *choice of* λ (see [12] and the references therein). However, in practice an optimal value of λ is difficult to select by any of these methods, and usually the choice of λ resorts to semi-empirical means, [11].

It is known that super-resolution methods, such as root-MUSIC, have the capability of asymptotically achieving optimal performance. However, in practice, with limited number of samples, or with highly correlated signal components, the accuracy performance often degrades away from the theoretical lower bounds, due to *resolution* limitations, [13]. In recent works, [7, 14], it was found that the $\ell 1$-regularization method may offer higher resolution than the super-resolution methods. Moreover, the sparse regularization has the advantage of producing good accuracy even at low signal-to-noise ratios (SNR), i.e., it exhibits good *robustness to noise*, as it has been noted in [14]. In fact it has been proved (see [15] and the references therein) that there is a fundamental connection between *robustness and sparsity*. Specifically, if some disturbance is allowed into the transformation matrix $\boldsymbol{\Gamma}_S$ or the measurements vector $\boldsymbol{y}_{kl}$, finding the optimal solution in the worst case sense is equivalent to solving the problem in the $\ell 1$-regularization formulation, which imposes sparse solutions.

The $\ell 1$-regularization continuously shrinks the estimate elements toward 0 as λ increases, leading to sparse solutions. However, the $\ell 1$-regularization shrinkage results in a small bias in the non-zero elements of the estimate, since the estimation of these elements is based on the measurement fidelity term, [16]. Thus solving the problem of estimating $\boldsymbol{h}_{kl}$ from the measurements $\boldsymbol{y}_{kl}$, (5), by employing an $\ell 1$-regularization formulation, (6), may lead to a sub-optimal solution for the non-zero elements of the estimate. However, despite this downside, with a reasonable choice of λ, the $\ell 1$-regularization method still produces better TDOA estimation

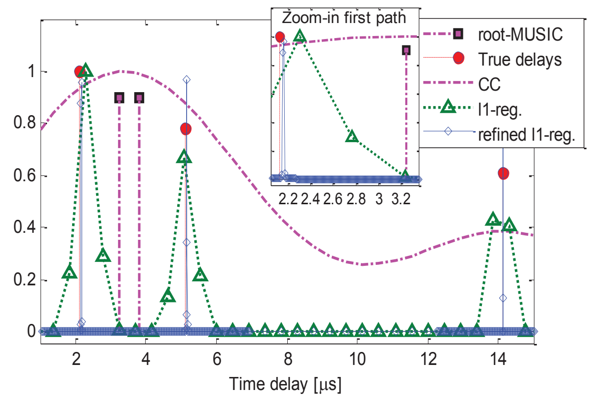

Fig. 1. True and estimated multipath components.

(and hence source localization) accuracy than conventional techniques, especially at low SNR, as demonstrated by the results presented in the next section. Moreover, the proposed method doesn't necessary require knowledge of the number of the multipath components, as root-MUSIC does.

When the power spectral density of the transmitted signal is flat across the frequencies of interest, $\boldsymbol{\Gamma}_S$ in (5) has the form of a DFT matrix. In this case, the sparse estimate can be found with fewer equations than in (6), reducing the required computational effort. A procedure for selecting a subset of equations among those in (6) and the sufficient number of equations in the subset to ensure that the solution is not altered, can be found in [17].

4. SIMULATION RESULTS

In this section, we present some simulation results to demonstrate the performance of the proposed $\ell 1$-regularization method. For a typical setup, we show significant improvement in localization accuracy, compared to conventional methods, such as cross-correlation and root-MUSIC. We considered a system in which a number $M = 8$ of sensors are approximately uniformly distributed around the source, on an approximately circular shape of radius 1 km. The source, which location is to be estimated, transmits a Gaussian Minimum Shift Keying (GMSK) modulated signal of bandwidth $B = 200$ kHz that is received by the M sensors through different multipath channels. For each sensor $k = 2, \dots M$, the TDOAs are measured relative to the chosen reference sensor, $l = 1$. The pulse shape is known at the sensors side and used to generate the auto-correlation of the transmitted signal. The wireless channels between the source and each of the sensors are modeled as (2). Specifically, a three-paths model is used, as in [4]. The first two paths are spaced well below the bandwidth resolution, while the separation between the second and the third path is higher, as it can be seen in Fig. 1. The only exception is the reference sensor assumed to be an AWGN channel, i.e., no multipath. The simulation scenario also employs the same noise level across sensors. For $\ell 1$-regularization, the regularization parameter λ is chosen by semi-empirical means, as motivated in Sec. 3.2. Originally, when solving problem (6), a number of 500 received symbols are sampled 8 times the Nyquist rate. An oversampling factor of $\times 50$ is used for grid refinement as explained in Sec. 3.1.

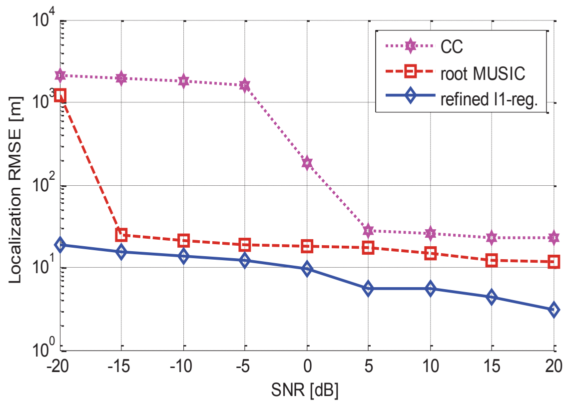

Fig. 2. Source localization accuracy in noise.

Fig. 1 shows the time-delays of the multipath components and their estimates at one of the sensors, for a received SNR of 15 dB per sample. The TDOA is given by the earliest of these components. The estimate by the $\ell 1$-regularization approach with grid refinement is the closest to the true delays, when compared to the other methods considered. The result of the $\ell 1$-regularization without grid refinement is visibly biased due to the limited sampling rate. The CC and the root-MUSIC estimates show significantly larger errors particularly for the two close paths. The TDOA estimates are transformed, by multiplication with the known signal propagation speed, into range difference information for constructing a set of hyperbolic equations. The solution provided by these equations is the estimated location of the source, relying on the knowledge of the sensors locations. In the literature, several methods can be found for solving the hyperbolic equations. We adopt here a recent approach that involves convex relaxation techniques supported by standard, efficient semidefinite programming (SDP) [18].

Fig. 2 illustrates the localization performance, in terms of root mean square error (RMSE) against SNR, of the proposed methods in the aforementioned scenario. The RMSE is obtained from Monte Carlo simulations with 100 runs per SNR value. The plot shows better accuracy of the proposed method over CC and root-MUSIC, at both high and low SNR. At high SNR, given the low separation between the first two multipaths, both CC and root-MUSIC provide biased estimates due to their limited resolution capabilities, though root-MUSIC is better. The reason for the better accuracy at low SNR is that $\ell 1$-regularization is robust to noise, as discussed in Sec. 3.2.

5. CONCLUSIONS

A method for source localization via TDOA estimation in multipath environments was developed. The sparsity of the channels is exploited and a grid refinement procedure was formulated to improve the resolution of the TDOA estimation. The proposed technique compares favorably to the conventional cross-correlation and root-MUSIC techniques, in terms of TDOA and source location accuracy estimation. For dense multipath environments the proposed method may succeed where conventional methods fail to resolve closely separated components. Therefore it is suitable for applications like source localization in multipath. Moreover, simulation results confirmed the noise robustness of the method.

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