Wireless Localization using Time Difference of Arrival in Narrow-Band Multipath Systems

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Abstract — **The problem of estimating the location of a source is addressed based on the time difference of arrival (TDoA) of the first multipath component of the signal at different sensors with known locations. A maximum likelihood (ML) approach is adopted, followed by the presentation of some two-steps techniques. The TDoA estimation is carried out using a correlation technique and a super-resolution method – root multiple signal classification (MUSIC). The source is relatively narrowband and it operates over a wireless channel with a dense multipath environment following the COST-207 channel model. The performance of the source location techniques is evaluated in terms of the root mean square error (rMSE) of the transmitter's position for given placements of the sensors. A precision contour-map provides a graphic, two-dimensional representation of the source location accuracy as a function of the source location and the location of the sensors.**

I. INTRODUCTION

The localization of a signal source in a communications or sensor networks is a classical problem, but it keeps being addressed by researchers due to changing requirements in terms of the channel over which the propagation takes place and localization accuracy. For example, in the USA, it is required now by the Federal Communications Commission (FCC) that the wireless service providers must report the call initiating mobile station (MS) location to an Emergency 911 (E-911) at the public safety answering point with an accuracy of 100 meters for 67% of all wireless E-911 calls. It is still expected that the required precision will be higher. But accurate localization is also desirable in many other applications [1], [2].

Localization techniques of wireless sources can be viewed as falling into two main categories namely, mobile based (or forward link) localization systems, and network based (or reverse link) localization systems [1]. In the first case, the MS (serving as a receiver) determinates its own location by measuring the signal parameters of an external system such as the cellular system it operates on or the global positioning system (GPS). In the second case, the system determinates the position of the MS (as transmitter) by measuring its signal parameters at the base stations (receiving sensors). The sensors measure the received signal and relay it to a central site for processing and estimation of the transmitter location. The technique relies on existing networks (e.g., cellular or wireless local area networks - WLAN). Network-based systems have the advantages of lower cost, size and battery consumption at the mobile device over the mobile-based systems. Also, in the GPS case, the mobile device needs signals from at least four satellites of the current network of 24 GPS satellites, albeit a hybrid method based on both GPS technology and the cellular infrastructure can also be used. Generally speaking, the GPS-based approach has a relatively higher accuracy, but it degrades in urban environments. All these considerations serve as motivation to seek improvements in network-based techniques for source localization.

II. NETWORK BASED LOCALIZATION USING TDOA

With the network-based methods for source localization, the processing is performed based on some properties of the signal received by the sensors [1], such as angle of arrival, signal strength, time of arrival, time difference of arrival, and combinations of these leading to hybrid techniques. Using these properties, the actual source location is computed by triangulation. The angle of arrival (AoA) (or direction of arrival (DoA)) method involves measuring angles of the source as seen by several sensors; the signal strength (SS) technique calculates the distance measuring the energy of the received signal; the time of arrival (ToA) procedure is based on measurements of travel time of the signal converted into distance, while the TDoA is different from ToA by utilizing a reference sensor. These methods can all be used depending on specific applications and environments, each of them having their own advantages and drawbacks: e.g., the AoA method requires antenna arrays at each sensor, which make it costly; for SS the channel (path-loss) model needs to be known, while ToA requires synchronization with the source clock.

In this paper we study network-based localization using TDoA. The source localization can be achieved either in one or two steps. In the case of a single-step, the location of the transmitter is estimated directly applying the maximum likelihood (ML) approach and making use of the TDoA calculation. With the two-steps approach, the goal of the first step is to estimate the TDoAs. The second step computes the source location utilizing the TDoA values already available.

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III. SINGLE-STEP LOCALIZATION

In this section we present a signal model for the source localization problem over the multipath channel, followed by a simple derivation of the ML approach for location estimation in the case of single-path propagation environment. Start, by assuming a source located at $\boldsymbol{X}_0 = (x_0, y_0)$. The signal transmitted by the source is collected by M sensors placed at arbitrary coordinates $\boldsymbol{X}_k^r = (x_k^r, y_k^r)$, $k = 1,..,M$. Then the complex valued signal received at sensor k is given by

$$r_k(t) = \sum_{q=1}^{L} A_{kq} s\left(t - \tau_k(\boldsymbol{X}_0)\right) + w_k(t), \qquad (1)$$

where A_{kq} is an unknown signal amplitude induced by the channel, L is the number of the channel's multipaths, $s(t)$ is the waveform transmitted by the source, $\tau_k(\boldsymbol{X}_0)$ is the time delay between the source and the sensor, and the term $w_k(t)$ is additive white Gaussian noise (AWGN) with variance σ_w^2, $w_k(t) \sim \mathcal{N}(0, \sigma_w^2)$. The propagation delay is related to the locations of the source and the sensor through

$$\tau_k(\boldsymbol{X}_0) = \frac{1}{c}\sqrt{\left(x_k^r - x_0\right)^2 - \left(y_k^r - y_0\right)^2}, \qquad (2)$$

where $c \simeq 3 \cdot 10^8\ m/s$ is the speed of light.

In the following, we limit discussion to the case of single-path propagation, making $L = 1$ and $A_{kq} = A_k$ in (1). This will enable us to get insight into the result. For simplicity, we also assume that the unknown amplitudes of the received signal are the same at all the sensors, $A_k = A$, $k = 1,..,M$, and that the waveform $s(t)$ is known at the receivers. The energy of the transmitted waveform is normalized to $\int \left|s(t)\right|^2 = 1$, such that the signal-to-noise ratio is $SNR = 1/\sigma_w^2$. The frequency domain representation of the received signal is given by

$$R_k(f) = A \cdot S(f) e^{-j2\pi f \tau_k(X_0)} + W_k(f), \qquad (3)$$

where $W_k \sim \mathcal{N}(0, \sigma_w^2)$. Consider now the signal received at another sensor, say l. We seek to express the received signal not in terms of $\tau_k(\boldsymbol{X}_0)$ and $\tau_l(\boldsymbol{X}_0)$, but rather in terms of TDoAs $\Delta_{kl}(\boldsymbol{X}_0) \triangleq \tau_k(\boldsymbol{X}_0) - \tau_l(\boldsymbol{X}_0)$. Define $Y_{kl}(f)$ as

$$Y_{kl}(f) \triangleq R_k(f) R_l^*(f) = B\left|S(f)\right|^2 e^{-j2\pi f \Delta_{kl}(X_0)} + Z_{kl}(f), \qquad (4)$$

where the terms of squared noise are neglected and $Z_{kl} \sim \mathcal{N}(0, 2B\sigma_w^2)$ with $B \triangleq |A|^2$, implying a probability density function (PDF) of $Y_{kl}(f)$ at a particular value of f $f\left(Y_{kl}(f) \middle| B, \boldsymbol{X}_0\right)$ of the form

$$\frac{1}{\sqrt{4B\pi\sigma_w^2}} \exp\left(-\frac{\left|Y_{kl}(f) - B\left|S(f)\right|^2 e^{-j2\pi f \Delta_{kl}(X_0)}\right|^2}{2B\sigma_w^2} \right). \qquad (5)$$

Considering a specific value of SNR, the likelihood function for a source at coordinates $\boldsymbol{X}$ is the PDF of $Y_{kl}(f)$ given the unknown parameters [6]. After some simple mathematical iterations, including the construction of the vector $\boldsymbol{Y}(f)$ consisting of terms $Y_{kl}(f)$ such that no index is repeated twice, taking *log* of the ratio of likelihoods and preserving only terms dependent on the coordinates, we obtain the test statistic

$$L(\boldsymbol{X}) = \sum_{k \in K, l \in L} \mathrm{Re}\left\{ \int_F Y_{kl}(f) \left|S(f)\right|^2 e^{-j2\pi f \Delta_{kl}(X)} df \right\}. \qquad (6)$$

where K and L contain the indices respectively k and l and F is the frequency range of the transmitted signal. We let the assignments of indices to the sets and pairings *k*, *l* be arbitrary.

Applying the Parseval's relation

$$\int_F Y_{kl}(f) \left|S(f)\right|^2 e^{-j2\pi f \Delta_{kl}(X)} df = \int y_{kl}(t) \rho\left(t - \Delta_{kl}(\boldsymbol{X})\right) dt, \qquad (7)$$

where $\rho(t) \triangleq s(t) * s(t)$, and "*" denotes convolution, the test statistic becomes

$$L(\boldsymbol{X}) = \sum_{k \in K, l \in L} \mathrm{Re}\left\{ \int y_{kl}(t) \rho\left(t - \Delta_{kl}(\boldsymbol{X})\right) dt \right\}. \qquad (8)$$

After computing first the TDoA $\Delta_{kl}(\boldsymbol{X})$ for all the pairs of sensors, the estimated coordinates $\widehat{\boldsymbol{X}_0}$ are found by choosing from all possible locations $\boldsymbol{X}$ the one that maximizes $L(\boldsymbol{X})$.

Further insight can be obtained by rearranging the terms of the integral in $L(\boldsymbol{X})$,

$$\int_{-\infty}^{\infty} R_k(f) S(f) e^{-j2\pi f \Delta_{kl}(X)} R_l^*(f) S^*(f) df. \qquad (9)$$

Denote $U_k(f) = R_k(f) S(f)$. The time domain correspondent $u_k(t) = \int_{-\infty}^{\infty} r_k(\lambda) s(t - \lambda) d\lambda = r_k * s(t)$ is viewed as the product of passing the received signal $r_k(t)$ through a matched filter to the known signal $s(t)$. The Parseval's relation now gives the time domain representation of integral (9) as $\int_{-\infty}^{\infty} u_k\left(t - \Delta_{kl}(\boldsymbol{X})\right) u_l^*(t) dt$ that leads to the nice interpretation that the ML formula is based on the shifted cross-correlation between match-filter outputs at each sensor.

IV. TWO-STEPS LOCALIZATION

The single-step ML approach has the advantage of a very good performance, but it is impractical because of the high computational effort required. One alternative is an indirect approach in which the location of the source is determined using TDoA estimates. The TDoA approach localizes the source on a hyperboloid with a constant range difference between the two sensors. This range difference is given by the expression $\tau_k(\boldsymbol{X}_0)-\tau_l(\boldsymbol{X}_0)$, where $\tau_i(\boldsymbol{X}_0)$, $i\in\{k,l\}$ is of form (2). Since the source can occupy only one point on the hyperbolic curve, using pairs of sensors and substituting the TDoAs Δ_{kl} estimated at the first step, we can find the location of the source with good precision. In the literature, there are several methods for solving the hyperbolic equations, see [1], [5].

For the estimation of the TDoAs, which is the first step, one natural approach is using the ML estimation in a similar form to that presented in the previous section. This time however the maximization is not performed for all the possible locations of the source, but for all the possible delays TDoA. It has been shown in [3] that for single path channel models this approach is equivalent to applying the generalized cross-correlation (GCC) technique with a Hannan-Thomson (HT) processor. This takes the received signals r_i, filters them by some function $H_i(f)$ specified in [3], v_i, $i\in\{k,l\}$ being the signals obtained after filtering. Then it takes the cross-correlation $R_{v_k v_l}$ of the results and searches for its maxima. The corresponding time lag represents the TDoA. The cross-correlation $R_{v_k v_l}(\tau)$ is $\int_{-\infty}^{\infty}\psi_g(f)G_{r_k r_l}(f)e^{j2\pi f\tau}df$, where $\psi_g(f)=H_k(f)H_l^*(f)$ is the HT processor and $G_{r_k r_l}(f)$ is the cross power spectral density function (the Fourier transform of the cross-correlation $R_{r_k r_l}(\tau)$) of the received signals $r_k(t)$ and $r_l(t)$. In practice, instead of the actual cross-correlation $R_{r_k r_l}$, an estimate is obtained from the finite observations r_k and r_l.

Aside from HT, other processors $\psi_g(f)$ have been suggested in the literature and tested for multipath channel models too, including the simple cross-correlator (CC), which assumes $\psi_g(f)=1$. The CC has the advantage of simple implementation, but unfortunately, it may lead to relative large biases, especially when it is used in narrow-band systems operating in a dense multipath environment. On the other extreme, the ML estimator is asymptotically optimal (achieves CRLB bound, which by definition is the lower limit of the variance of an unbiased estimate [4], asymptotically as SNR or the number of signal samples goes to infinity), but is computationally complex involving a multi-dimensional search.

Another option available for TDoA estimation is the application of super-resolution techniques. The basic idea is to estimate the noise subspace through eigen-decomposition, and then to estimate the signal parameters by utilizing the fact that the signal vector is orthogonal to the noise subspace. Based on this, an objective function is constructed such that its first largest, say L, peaks offer a way to find the unknown parameter of interest, the TDoA, in our case. Root-MUSIC is one such technique that seems to offer good performance, especially at low SNR. In this case, the objective function takes the form of a polynomial, and it is necessary to find the L roots with the largest magnitude (closest to the unit circle) [6]. Root-MUSIC is computationally attractive since it employs only a one-dimensional search, compared to the ML estimation which requires a multi-dimensional search.

V. SIMULATIONS AND RESULTS

For simulations, we considered a system in which a number of $M\in\{4,6,8\}$ sensors are evenly distributed on a circle of radius 1400 meters and having the center at coordinates $(x_c=0, y_c=0)$. A transmitter placed at an unknown location sends a Gaussian minimum shift keying (GMSK) modulated signal of bandwidth $200kHz$ that is received by the M sensors in different forms according to the wireless propagation channel model. Each version of the signal received by sensor k is delayed by τ_k. The TDoAs are measured relative to a chosen reference sensor, say $l=1$, such that the difference in time of arrivals between sensors k and l is $\Delta_{kl}=\tau_k-\tau_1$. The wireless channel between the source and each sensor follows the model (1). Specifically, it follows the COST-207 model (i.e. multipath with $L=6$ Rayleigh faded rays). The only exception is the reference sensor which was assumed to be an AWGN channel, i.e., no multipath. The simulation scenarios employ the same mean SNR across the sensors. The localization methods were carried out in two steps: (1) estimate the TDoA of first arrivals using the CC or root-MUSIC algorithm; (2) using the TDoA estimates, determine the source location. As a performance measure, we used the rMSE of the location.

Fig. 1 illustrates the performances of the CC and root-MUSIC techniques in a scenario with $M=4$ sensors, the source being located at coordinates $\boldsymbol{X}_0=(1306\text{ m},\ 541\text{ m})$. The plot shows that the root-MUSIC technique has better performance than CC even at low SNR.

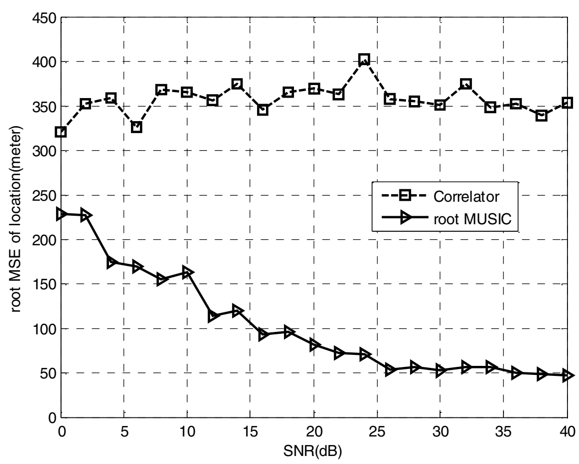

Figure 1. Localization for 4 sensors, source point at x_0=1306, y_0=541

The estimation bias of the CC techniques is manifested by the error floor that is independent of SNR. In contrast, the performance of root-MUSIC is asymptotically unbiased and its rMSE decreases as the SNR increases, since it can resolve more and more multipaths.

Fig. 2 compares the performance of three different scenarios with respectively, 4, 6 and 8 sensors. All three scenarios assume the source at coordinates $\boldsymbol{X}_0 = (435 \text{ m}, 180 \text{ m})$, and use the root-MUSIC algorithm for TDoA estimation. One can observe that by increasing the number of sensors, the root-MSE decreases, but the accuracy gained by increasing from 6 to 8 sensors is lower than the accuracy gained by increasing from 4 to 6 sensors.

For the third simulation scenario, we fixed the SNR to 20 dB, used 6 sensors and considered a grid of possible locations of the source. For each such possible location $\boldsymbol{X}_0$, we applied the localization technique with root-MUSIC for TDoA estimations. The contour map representing rMSE values is shown in Fig. 3. Due to the symmetrical configuration chosen, the map is symmetrical with respect to the horizontal axis. Such a representation is practically useful as it provides a graphic, two-dimensional illustration of the estimation accuracy as a function of the source location and the location of the sensors.

VI. Conclusions

The paper begins by briefly reviewing the main approaches in the localization of a signal source. Then we considered the TDoA estimation approach and derived a simple single-step ML estimator in the case of AWGN propagation channel which led to the interpretation that the ML formula is based on the shifted cross-correlation between match-filter outputs at each sensor. Finally we discussed the two-steps localization approach and showed simulation results in a multipath environment over some scenarios for the CC and root-MUSIC algorithms. The root-MUSIC showed an improved performance over the CC as expected, but increasing the number of sensors from 6 to 8 does not bring the same accuracy gain as increasing from 4 to 6, at least in the case of root-MUSIC algorithm. Finally an accuracy contour map for a given scenario was presented.

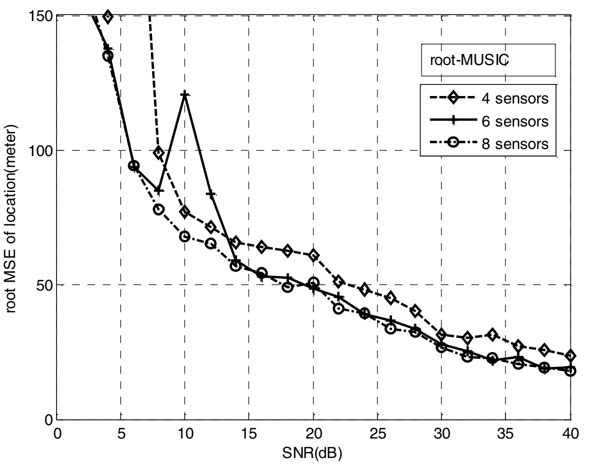

Figure 2. Localization for 4, 6, 8 sensors, source at x_0=435, y_0=180

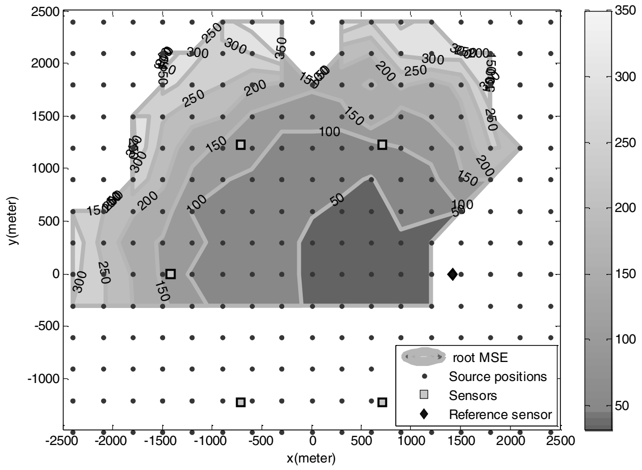

Figure 3. Root MSE contour for 6 sensors and different positions of the source, SNR=20dB, root-MUSIC

When comparing our results to the FCC requirement that the wireless service providers must report the call initiating MS location to an E-911 at the public safety answering point with an accuracy of 100m for 67% of all wireless E-911 calls, we conclude from Fig. 1 that usage of the CC algorithm produces insufficient accuracy (about 300m for the chosen scenario). The root-MUSIC algorithm instead gives better accuracy than required by FCC in the scenarios considered for both Fig. 1 and 2, for sufficiently high SNR. One can also observe that the accuracy of localization is highly dependent on the placement of the source relative to the sensors. For instance, from Fig. 3 it is easy to read the upper bound of the accuracy reached in the localization of a source from a specific location relative to the 6 sensors, using root-MUSIC algorithm for TDoA estimation, when an SNR of 20dB is available.

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