On the Stackelberg Equilibrium of Total Weighted Squared Correlation in Synchronous DS-CDMA Systems: Algorithm and Numerical Results

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***Abstract*— We characterize the Total Weighted Squared Correlation (TWSC) used in uplink synchronous code division multiple access (S-CDMA) as a payoff function in the context of the game theoretic framework. We proved that the value of the game for two players is a saddle-point corresponding to the minimum of TWSC [1]. The optimal solution is the Stackelberg equilibrium and is achieved by players choosing generalized Welch Bound Equality (WBE) sequences as spreading signatures. We propose an algorithm of minimizing TWSC in the presence of multipath and colored noise when channel state information is known perfectly at both transmitter and receiver. A comparison of our method in terms of BER (bit error rate) and water-filling procedure with the previous known methods is also given.**

I. Introduction

By characterizing the Total Weighted Squared Correlation (TWSC) used in uplink synchronous code division multiple access (S-CDMA) [5], [9], [11] as a payoff function in the context of the game theoretical framework [13], [14-17] we established that the optimal solution is a Stackelberg saddle point and we derived the mathematical framework to reach it. In this paper we present a practical iterative algorithm to construct the optimal solution (Section II) for arbitrarily varying channel with colored noise. Then we compare our proposed method with other methods by means of three experiments (Section III). Finally, conclusions are drawn and future work is envisaged (Section IV).

II. The Practical Algorithm

Assume channel matrix is known both at transmitter and receiver. We minimize TWSC in a multipath channel considering a received vector in $N+L-1$ dimensional space (N is the spreading game and L is the number of multipaths) by controlling the transmitted sequence in N dimensional space [2], [9]. Let $\mathbf{G}_R$ be the Gram matrix of received spread sequences and $\mathbf{\Sigma}$ the covariance matrix of colored noise. We generalize the results obtained in [8] and [9] minimizing the Frobenius norm of the matrix $\mathbf{P}^{1/2}\mathbf{G}_\mathbf{R}\mathbf{P}^{1/2}+\mathbf{\Sigma}$ by controlling its eigenvalues. At each step these eigenvalues are modified using a majorization constraint starting with the maximum one. The minimum eigenvalue is always kept constant and is modified only in the last step. The proposed method is a generalization of the uniform good property described in [12] and it achieves the minimization of TWSC in $N+L-2$ steps. The corresponding spreading signature sequences are obtained decentralized at each step by using the algorithm given in [11] for uplink multiple cell CDMA systems and it is based on the inverse eigenvalue problem.

It is known that so called iterative multiuser water filling method [10] achieves the global optimum of the sum capacity in a fully decentralized way and this coincides with a Nash equilibrium since each user water fills the noise plus interference caused by the others. The method given in [10] is very robust after the first iteration. The method proposed in this paper is not so robust after the first iteration, but it controls exactly the number of steps. Both methods perform optimization in the space of positive semidefinite matrices and do not depend on the initial starting point. However, for a large dimensional signal space and a large number of users it is desirable to let one or all users updating their sequence simultaneously in a finite number of steps.

Given a received normalized correlation matrix $\mathbf{G}_\mathbf{R}$ (the initial point of recursion) in the presence of a colored noise matrix $\mathbf{\Sigma}$, such that the eigenvalues of TWSC are arranged in decreased order $\nu_i \geq \nu_{i+1} \geq 0$, we have $TWSC(\mathbf{S}_\mathbf{R})=\sum_{i=1}^{K}\nu_i^2$ and $\sum_{i=1}^{K}\nu_i = p_{tot}+trace(\mathbf{\Sigma})$ (K is the total number of users). By Schur theorem for Gram matrix $\mathbf{G}_\mathbf{R}$ we have the following majorization condition satisfied:

$$\underbrace{(1,..,1)}_{K} \prec \left(\underbrace{\frac{K}{N+L-1},\ldots,\frac{K}{L+N-1}}_{N+L-1},\underbrace{0,\ldots,0}_{K-(N+L-1)}\right) \prec \prec(\nu_1,\nu_2,\ldots,\nu_K) \qquad (1)$$

Obviously, if matrix $\mathbf{G}_\mathbf{R}$ is chosen at random, $TWSC(\mathbf{S}_\mathbf{R}) \geq \geq K^2/(N+L-1)$ holds. Our iterative method for minimizing TWSC and obtaining the corresponding WBE sequences [3] is reduced to constructing the matrices $\mathbf{W}_i$ and $\mathbf{\Sigma}_i$ given by (5) and it consists in the following steps:

1. *Input*: the number of users K, the spreading gain N, ($K > N$) the number of multipaths L, ($K > N + L - 1$), the vector power of the users $\mathbf{p} = [p_1 \geq p_2 \geq, ..., \geq p_K]$ and noise densities such that $eig(\mathbf{\Sigma}) = [\sigma_1^2 \leq \sigma_2^2 \leq, ..., \leq \sigma_{N+L-1}^2, 0, ..., 0]$ (in order to check with the work given in [2], [6]). Determine the oversized users as in [6]. If there are M oversized users use (6). If all users are nonoversized then (1) becomes:

$$(p_1 + \sigma_1^2, p_2 + \sigma_2^2, ..., p_K + \sigma_K^2) \prec \\ \prec \begin{pmatrix} \dfrac{p_{tot} + trace(\Sigma)}{N+L-1}, \dfrac{p_{tot} + trace(\Sigma)}{N+L-1}, ... \\ ..., \dfrac{p_{tot} + trace(\Sigma)}{N+L-1}, 0, ..., 0 \end{pmatrix} \prec (\nu_1, \nu_2, ..., \nu_K) \quad .(2)$$

2. Given a random weighted Gram matrix $\mathbf{W}$ of order $K \times K$ and rank N calculate $eig(\mathbf{W})$. Algorithms to generate random weighted Gram matrices are well known; see for example "gallery" Higham test matrices provided in Matlab. Find a permutation matrix $\mathbf{Q}$ such that the matrix $\mathbf{W}_0 = \mathbf{Q}^T \mathbf{W} \mathbf{Q}$ has eigenvalues $eig(\mathbf{W}_0) = \mathbf{\Lambda}_0 = (\lambda_1, \lambda_2, ..., \lambda_K)$. Find an orthogonal matrix $\mathbf{U}_0$ satisfying $\mathbf{W}_0 = \mathbf{U}_0^{\mathbf{T}} diag(\mathbf{\Lambda}_0) \mathbf{U}_0$ using the *Algorithm* in [11]. Assuming that $\mathbf{W}$ and $\mathbf{\Sigma}$ commute the same procedure is valid for $\mathbf{\Sigma}$, so we get $\mathbf{\Sigma}_0 = \mathbf{U}_0^{\mathbf{T}} diag(\boldsymbol{\sigma}_0) \mathbf{U}_0$ where $\boldsymbol{\sigma}_0 = eig(\mathbf{\Sigma}_0) = [\sigma_1^2, \sigma_2^2, ..., \sigma_{N+L-1}^2, 0, .., 0]$. Let $\mathbf{v}_0 = \mathbf{\Lambda}_0 + \boldsymbol{\sigma}_0$.

3. For $1 \leq n \leq N + L - 2$ construct majorization eigenvalue set

$$\mathbf{v}_n = \begin{pmatrix} \underbrace{\dfrac{p_{tot} + trace(\Sigma_0)}{N+L-1}, ..., \dfrac{p_{tot} + trace(\Sigma_0)}{N+L-1}}_{n}, \\ \nu_1 + \nu_2 + ... + \nu_{n+1} - n\dfrac{p_{tot} + trace(\Sigma_0)}{N+L-1}, \\ \nu_{n+2}, ..., \nu_{N+L-1}, \underbrace{0, ..., 0}_{K-(N+L-1)} \end{pmatrix}. \quad (3)$$

At each step n the old eigenvalues set is replaced by a new one using the majorization constraint $\mathbf{v}_{n+1} \prec \mathbf{v}_n$. In the last step for $n = N + L - 2$ we have:

$$\mathbf{v}_{L+N-1} = \left(\frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, ..., \frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, 0, ..., 0 \right). \quad (4)$$

4. Construct matrices

$$\mathbf{W}_i + \mathbf{\Sigma}_i = \mathbf{U}_i^T \left[\mathbf{U}_0 (\mathbf{W}_0 + \mathbf{\Sigma}_0) \mathbf{U}_0^T - diag(\mathbf{v}_0 - \mathbf{v}_i) \right] \mathbf{U}_i \quad (5)$$

which is the desired eigenvalue set.

If there are M oversized users in the system then the majorization constraint given by (1) becomes:

$$\left(p_1 + \frac{trace(\mathbf{\Sigma})}{N+L-1}, p_2 + \frac{trace(\mathbf{\Sigma})}{N+L-1}, ..., p_K + \frac{trace(\mathbf{\Sigma})}{N+L-1} \right) \prec \\ \prec \left(p_1 + \sigma_1^2, p_2 + \sigma_2^2, ..., p_M + \sigma_M^2, \underbrace{\nu, ..., \nu}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)} \right) \prec \quad (6) \\ \prec (\nu_1, \nu_2, ..., \nu_K)$$

where: $$\nu = \frac{\sum_{l=M+1}^{K} (p_l + \sigma_l^2)}{N+L-1-M}. \quad (7)$$

For $1 \leq l \leq N + L - 1 - M$ (20) becomes:

$$\boldsymbol{\mu}_l = \begin{pmatrix} p_1 + \sigma_1^2, ..., p_M + \sigma_M^2, \\ \nu_1 + \nu_2 + ... + \nu_{l+1} - \sum_{i=1}^{l} (p_i + \sigma_i^2), \underbrace{\nu, ..., \nu}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)} \end{pmatrix} \quad (8)$$

and repeat steps 3) and 4). In the last step we have:

$$\boldsymbol{\mu}_{L+N-1} = (p_1, ..., p_M, \underbrace{\nu, ..., \nu}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)}) \quad (9)$$

which is the desired eigenvalue set.

III. NUMERICAL RESULTS

<u>*Experiment 1*</u>. Consider $K = 8, N = 5, L = 2$ in order to compare our results in the absence of multipath with the ones given in [2, Fig.1] and [3]. All users have unitary power and the noise variance is $\boldsymbol{\sigma} = [0.023\ 0.947\ 1\ 1.005\ 1.010\ 1.015\ 0\ 0\ 0\ 0]$. We start with a random weighted Gram matrix and after a permutation the following eigenvalue vector gives the eigenvalues of initial point of iterations: $\mathbf{\Lambda}_0 = [3.673\ 2.426\ 2.316\ 2.305\ 2.150\ 2.130\ 0\ 0]$. The results given by the proposed method are plotted in Fig.1 against the ones of the TSC minimization algorithm [2], [3]. In only 5 steps (up-dates) the proposed method produces an optimal spreading signature set that reaches the Welch bound [12].

The proposed method is convergent and stable for any channel realization in contrast with the results of TSC minimizing algorithm for Channel B [2, Fig.2]. We also compare our method with the algorithm proposed in [3] from the BER perspective. The results are plotted in Fig. 2 where we used the results deduced in the Appendix for both procedures. The improvements of our methods are substantial for minimizing TSC in the range $TSC \in [10.66 \quad 12]$. This result is expected since from Fig.1, for $TSC = 12$, by using the proposed method, the optimal signature set is only a step away from the Welch bound while the algorithm proposed in [2], [3] requires 11 additional updates. The main advantage of the proposed method is relevant in the case of the oversized users. Such results are not reported in [2].

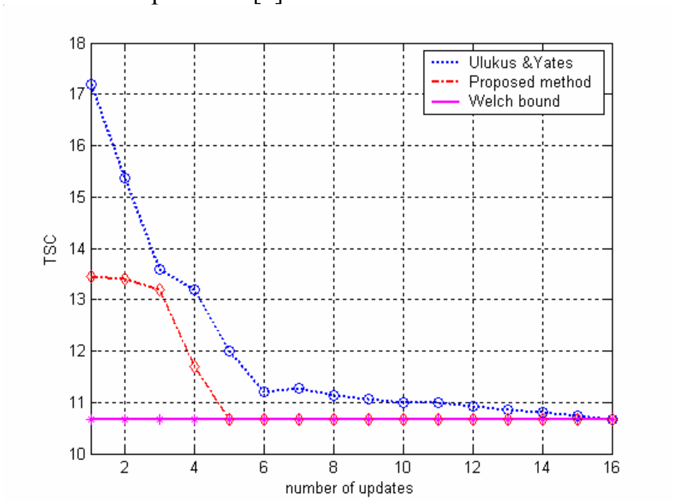

Figure 1. Convergence of the *TSC* in function of number of updates necessary for K=8, N=5, and L=2 to meet the Welch bound

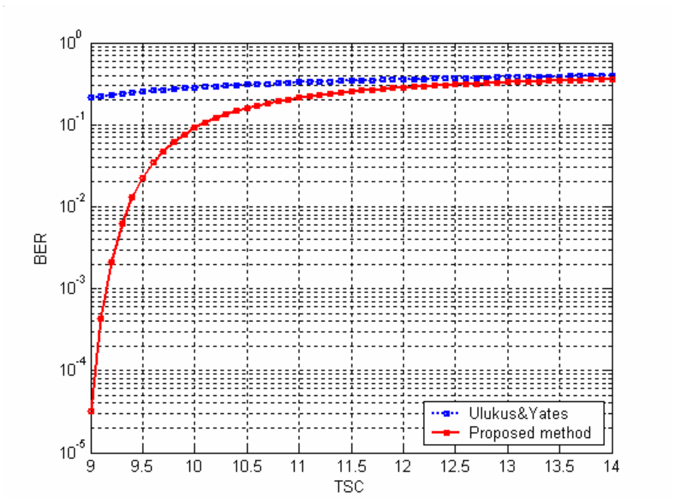

Figure 2. Bit error probability versus *TSC* for K=8, N=5, and L=2

<u>*Experiment 2.*</u> In this experiment we compare the sum capacity maximization obtained by the proposed method with the sum capacity of parallel Gaussian channels (the maximum sum of rates per unit channel at which all the information can be transmitted reliably in each of the channels). Considering the same data as in *Experiment 1* we can apply water filling method for three users. The water level in this numerical example is $0.5(trace(\mathbf{P})+\sum_{i=1}^{3}\sigma_i^2)$ and the results are plotted in Fig.3. It is more relevant in this figure that it is not necessary to obtain an increased sum capacity after each step, as it was required in previous works [7]. For example, in the first three steps the sum capacity is decreasing and after that it increases towards its maximum value in the next two steps.

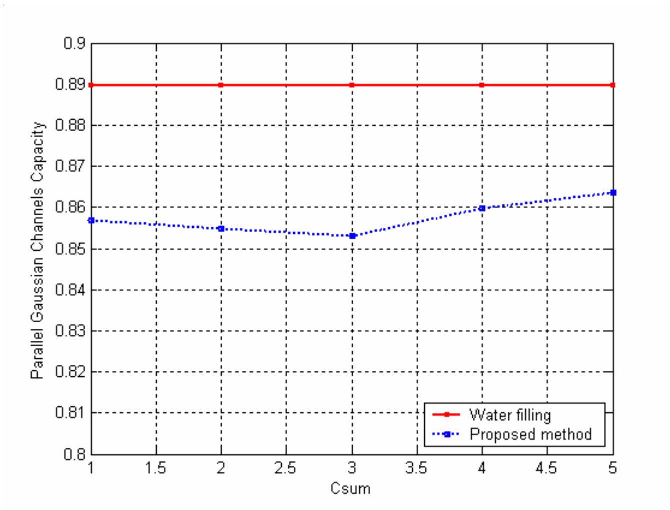

Figure 3. Sum capacity under proposed method versus water filling method

IV. Conclusions and Future Work

In this paper we have focused on uplink symbol-synchronous CDMA systems in the presence of colored noise. The extension of our results to the asynchronous situation and considering colored noise is interesting and also an important open problem. Our current efforts are directed towards solving this important open question. The extension to multiple base stations is a challenging task.

Using a game theoretical approach we defined the Total Weighted Squared Correlation as an extensive game payoff and we derived the optimal strategy for this game. By using majorization theory we derived the saddle point and this coincides with the minimum TWSC. A new method of reducing TWSC is provided. The solution of Stackelberg equilibrium corresponding to TWSC is obtained by the generalized WBE sequences.

Appendix

At the transmitter side, the total interference in the system is defined as $I=\sum_{\substack{i,j=1\\ i\neq j}}^{K}\left|<\mathbf{s}_i,\mathbf{s}_j>\right|^2$ and it is minimized in the case of WBE sequences to:

$$I_{\min}=\sum_{\substack{i=1\\ i\neq j}}^{K}\sum_{j=1}^{K}\left|<\mathbf{s}_i,\mathbf{s}_j>\right|^2=\frac{K^2}{N}-K\,. \qquad (10)$$

The interference $I(k)=\sum_{\substack{j=1\\ j\neq k}}^{K}\left|<s_k,s_j>\right|^2$ for <u>any</u> user k is minimized to the same value:

$$I_{\min}(k)=\sum_{\substack{j=1\\ j\neq k}}^{K}\left|<\mathbf{s}_k,\mathbf{s}_j>\right|^2=\frac{\frac{K^2}{N}-K}{K}=\frac{K}{N}-1\,. \qquad (11)$$

This is the so called uniform good property [12]. Note that, in general, the values of the particular interferences $I(i)$ and $I(j)$ for different users i and j are not equal since $\left|<\mathbf{s}_i,\mathbf{s}_j>\right|^2$ are different $\forall i,j=1,...,K,i\neq j$. However, the expected value of the single user interference can be obtained with:

$$E\left[I(k)\right]=\frac{\sum_{k\neq j}^{K}\left|<\mathbf{s}_k,\mathbf{s}_j>\right|^2}{K-1}=\frac{(K-N)}{N(K-1)}. \quad (12)$$

Considering (10) the BER for unitary power valued WBE sequences (real or complex) is the same for each user k and in the case of overloaded CDMA systems ($K>N$) it is given by:

$$\Pr(\hat{x}_k\neq x_k)=Q[(\sigma^2+I_{\min}(k))^{-1/2}]=Q[(\sigma^2+\frac{K}{N}-1)^{-1/2}]. \quad (13)$$

Consider now the case when all users have different powers and no user is oversized. The total power in the system is $p_{tot}=\sum_{i=1}^{K}p_i$. The Gram matrix associated has the optimum eigenvalue distribution vector given by $(\frac{p_{tot}}{N},...,\frac{p_{tot}}{N},\underbrace{0,...0}_{K-N})$, [9] and (11) becomes:

$$I_{\min}=\sum_{i=1}^{K}\sum_{j=1}^{K}p_ip_j\left|<s_i,s_j>\right|^2=\frac{p_{tot}^2}{N}-\sum_{i=1}^{K}p_i^2. \quad (14)$$

In order to obtain BER we want the above expression to be expressed in term of TWSC. Let's consider the following majorization relation that characterizes the minimum TWSC in this particular case:

$$\left(p_1,p_2,...,p_K\right)\prec\left(\underbrace{\frac{p_{tot}}{N},\frac{p_{tot}}{N},...,\frac{p_{tot}}{N}}_{N},\underbrace{0,...,0}_{K-N}\right). \quad (15)$$

Multiplying by K and simplifying by p_{tot} we obtain:

$$\left(\frac{Kp_1}{p_{tot}},\frac{Kp_2}{p_{tot}},...,\frac{Kp_K}{p_{tot}}\right)\prec\left(\underbrace{\frac{K}{N},\frac{K}{N},...,\frac{K}{N}}_{N},\underbrace{0,...,0}_{K-N}\right), \quad (16)$$

which corresponds to WBE sequences having the uniform good property (each column of Gram matrix has the same norm and it is associated to the interference of each user). Thus:

$$I_{\min}(k)=\frac{p_{tot}^2}{KN}-p_k^2=\frac{TWSC}{K}-p_k^2 \quad (17)$$

and it is the same for each user. Using (33) BER becomes:

$$\Pr(\hat{x}_k\neq x_k)=Q\left[(\sigma^2+\frac{TWSC}{K}-p_k^2)^{-1/2}\right]. \quad (18)$$

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