On the Stackelberg Equilibrium of Total Weighted Squared Correlation in Synchronous DS-CDMA Systems: Theoretical Framework

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Abstract—We characterize the Total Weighted Squared Correlation (TWSC) criterion used in uplink synchronous code division multiple access (S-CDMA) as a payoff function in the context of the game theoretical framework. By using the majorization theory we proved that the value of the game for two players is a saddle-point corresponding to the minimum of TWSC (Stackelberg equilibrium point). The optimal mixed strategies for transmitter and jammer are derived.

I. INTRODUCTION

With the explosive demand for high speed data services, mobile users in 4G systems will be serviced with data rates as high as 20 Mb/s. The future 4G-network architecture is expected to converge into a heterogeneous, all IP architecture, which includes different wireless access networks such as 4G CDMA cellular networks, wireless LAN, Bluetooth, and ultra-wideband systems. Integrating multiple subsystems into 4G networks brings about many challenges such as end-to-end quality of services (QoS), subsystem interworking and transmitter adaptation. In the future CDMA cellular systems, transmitter adaptation is one of the core technologies leading to power efficient wireless communication systems.

Typical physical-layer in adaptive multiuser detection for CDMA considers transmitter parameters like rate, power, spreading codes, error-correcting codes, spreading gain to be fixed [4]. Optimization is usually attempted at the receiver only. In recent years, more researchers have investigated transmitter optimization but usually in the context of rate optimization or power control. In many situations, the transmitter is constrained on its average power in order to limit the interference level of the system [6], [10].

One of the CDMA transmitter parameters that is largely ignored in adaptive systems is the spreading code. It is well known that the system capacity of 4G CDMA cellular systems is interference limited. With multiple-access interference reduction in mind, researchers have considered optimal spreading sequences for synchronous CDMA (S-CDMA) over AWGN channels when the number of users is greater than the spreading gain [7], [8], [11], [14], [15], [17], [30].

In this paper we characterize the Total Weighted Squared Correlation (TWSC) used in uplink S-CDMA [8], [13], as a payoff function in the context of the game theoretical framework [20], [22], [23]. We consider a varying channel [21], where the channel state can arbitrarily change from symbol to symbol during the transmission. When channel state information is available at the transmitter then optimal power allocation that achieves the information capacity is well known and in such a case the capacity is achieved by adapting the transmitted signal to a specific channel realization. In other words, the transmit signaling directions need to align their right singular vector of the channel matrix [16]. The transmit power has to be optimally allocated in a water filling fashion [18].

This paper is organized as follows: In Section II we present the game theoretical formulation and the system model. We define the value of the game as the minimum of TWSC in the context of rectangular games. The mathematical framework for the mixed strategies is the subject of the Section III. The conclusions and future work are drawn in Section IV.

II. GAME THEORETICAL FORMULATION AND SYSTEM MODEL

We will consider a game in which the payoff function (by which the results of the game is measured) is TWSC and the players are: the transmitter that selects the best signature sequences matrix in order to minimize TWSC and the jammer [22],[29] in the same cell that chooses the worst communication conditions to maximize TWSC. The unknowns of the game are the transmit covariances or the spreading signature sequences of the players. The problem can be viewed as a two player zero sum game: the transmitter is the minimizing player and the jammer is the maximizing player\textsuperscript{1}.

We formulate our problem as a strategic game (in general it has not a pure Nash equilibrium strategy) in a more general way as an extensive game in which there is an explicit description of the sequential structure of the decision problems. We consider the case in which the transmitter moves first and the jammer moves aware of the transmitter’s move. In such a case the solution is the Stackelberg equilibrium [24-27]. The concept of Nash equilibrium is unsatisfactory in extensive games since it ignores the sequential structure of the decision problems [23].

\textsuperscript{1} It is interesting to note that the same game under TWSC payoff function can be formulated for users in different cells that are received by the same base station. This scenario corresponds to the multicellular case that is an open problem being under research right now, and the extension of our results to this challenging case is left for future research.
Another payoff function that has been also considered is the mean-square error in [21] in the case of communication over a channel with an intelligent jammer [22] considering the covariance constraint for the worst additive noise [28]. In that case the Gaussian distribution was obtained as a saddle point solution. The extension to vector Gaussian arbitrarily varying channels was considered in [29] obtaining a saddle point given by water-filling solution for the jammer and the transmitter.

For our purposes we define a codeword $\mathbf{S} = [s_1, s_2, \ldots, s_k]$ of length $K > N$ as a real $N \times K$ matrix having signature sequences of length $N$ (spreading gain or the dimensionality of the signal space) as columns, which is selected by the transmitter. A jamming codeword of length $K$ is a $N \times K$ matrix $\mathbf{S}_j$ selected by the jammer. The data vector $\mathbf{d}^T = [d_1, d_2, \ldots, d_k]$ ($T$ means transposition) containing the information of all users $K$ is transmitted with the power $\mathbf{p}^T = [p_1, p_2, \ldots, p_k]$. Similarly, we consider, $\mathbf{p}_j$ and $\mathbf{d}_j$ as the jammer’s power and data, respectively. For two cells, we can express the received signal at the base station $i = 1, 2$ on the uplink DS-CDMA systems as:

$$r_i = \mathbf{S}_i \text{diag}(\mathbf{p}) \mathbf{d} + \mathbf{S}_j \text{diag}(\mathbf{p}_j) \mathbf{d}_j + \mathbf{z}_i$$

where $\mathbf{S}_i, \mathbf{S}_j, \mathbf{p}, \mathbf{p}_j, \mathbf{z}_i$ are the signature matrix of the users, the signature matrix of the jammer, the power of the users, power of the jammer and the noise, respectively, in cell $i$.

The following particular cases fit into the general model of (1):

i) The Gaussian interference channel for two users that are received by two receivers [18], [19];

ii) The Gaussian arbitrarily varying channels [21], for a transmitter and a jammer (both the transmitter and the jammer are subject to power limitations on transmitted power);

iii) The users in two different cells on the uplink DS-CDMA channels [17].

For any real $N \times K$ matrix $\mathbf{S}$ denote the (time-averaged) power of $\mathbf{S}$ by the averaged Frobenius norm:

$$P(\mathbf{S}) = \frac{1}{K} \| \mathbf{S} \|_F^2.$$  

The transmitter is assumed to have a constraint in its total power (a long term power constraint):

$$P_{\text{tot}} = \sum_{i=1}^K p_i.$$  

In the normalized case we will assume $P_{\text{tot}} = 1$.

Given a signature sequence set $\mathbf{S} = [s_1, s_2, \ldots, s_k]$, and the power matrix $\mathbf{P} = \text{diag}(\mathbf{p})$, then the TWSC of $\mathbf{S}$ in the absence of the noise is the sum of the squared magnitudes of all weighted inner products between the signatures (of length $N$) [1], [6-12]:

$$TWSC(\mathbf{S}) = \sum_{i=1}^K \sum_{j=1}^K p_i \langle s_i, s_j \rangle.$$  

TWSC is a global measure of the total interference level in a CDMA system [14], it is a weighted sum of the interference power “seen” by every user and it generalizes the Total Squared Correlation (TSC) criterion [30]. It is easy to verify that:

$$TWSC(\mathbf{S}) = \sum_{i=1}^K \sum_{j=1}^K p_i \langle s_i, s_j \rangle = \| \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \|_F^2 = \sum_{i=1}^K \lambda_i^2,$$  

where $\lambda_i$ are the eigenvalues of the weighted Gram matrix $\mathbf{W} = \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2}$ and $\mathbf{G}$ is the Gram matrix [3].

We shall now extend this definition so as to cover arbitrary rectangular games (i.e. rectangular games whose matrices have arbitrary numbers of rows and columns). Let us consider the rectangular game of order $m \times n$ whose matrix is:

$$\begin{bmatrix}
   \langle s_1, s_1 \rangle & \langle s_1, s_2 \rangle & \cdots & \langle s_1, s_n \rangle \\
   \langle s_2, s_1 \rangle & \langle s_2, s_2 \rangle & \cdots & \langle s_2, s_n \rangle \\
   \vdots & \vdots & \ddots & \vdots \\
   \langle s_n, s_1 \rangle & \langle s_n, s_2 \rangle & \cdots & \langle s_n, s_n \rangle 
\end{bmatrix}.$$  

By a mixed strategy for the player $P_1$ we mean an ordered $m$-tuple $S_n = [x_1, \ldots, x_n]$ of nonnegative real numbers satisfying the condition $\sum x_i = 1$; these numbers are the powers given by (3), which the player $P_1$ allocates to the users $1, 2, \ldots, m$. Similarly, by a mixed strategy for the player $P_2$ we mean an ordered $n$-tuple $S_n = [y_1, \ldots, y_n]$ of nonnegative real numbers satisfying the condition $\sum y_j = 1$.

If $P_1$ uses the mixed strategy $\mathbf{X} = [x_1, \ldots, x_n]$, and if $P_2$ uses the mixed strategy $\mathbf{Y} = [y_1, \ldots, y_n]$, then the mathematical expectation of $P_1$ is given by the following formula:

$$E(\mathbf{X}, \mathbf{Y}) = \sum_{j=1}^n \sum_{i=1}^m x_i y_j \langle s_i, s_j \rangle,$$  

which is the same definition of TWSC given by (4) and (5), but for the particular case $\mathbf{X} = \mathbf{Y} = \mathbf{p}^T$, $m = n = K$, and in the absence of noise. If it happens that for some $\mathbf{X}^* \in S_n$ and some $\mathbf{Y}^* \in S_n$ we have:

$$E(\mathbf{X}, \mathbf{Y}^*) \leq E(\mathbf{X}^*, \mathbf{Y}^*) \leq E(\mathbf{X}^*, \mathbf{Y}),$$  

for all $X$ in $S_m$ and all $Y$ in $S_n$, then we call $X^*$ and $Y^*$ optimal (mixed) strategies for $P_1$ and $P_2$, respectively, and we call $E(X^*, Y^*)$ the value of the game for $P_1$. If $X^*$ and $Y^*$ are the optimal strategies for $P_1$ and $P_2$, respectively, then the ordered pair $[X^*, Y^*]$ is a strategic saddle point.

Consider $X_i = [0, ..., 0, x_i, 0, ..., 0] = [0, ..., 0, 1, 0, ..., 0]$ whose $i$th component is replaced by the sum of all components of $X$. Then we write $E(i, Y) = E(X, Y) = \sum_{j=1}^{n} s_i' s_j' j' y_j$. Similarly, consider $Y_i = [0, ..., 0, y_j, 0, ..., 0] = [0, ..., 0, 1, 0, ..., 0]$ whose $j$th component is replaced by the sum of all components of $Y$. Also we write $E(X, j) = E(X, Y) = \sum_{i=1}^{n} s_i' s_j' i' x_i$.

We note that the expectation given by (7) can be written in the equivalent form:

$$E(X, Y) = \sum_{i=1}^{n} E(i, Y)x_i = \sum_{j=1}^{n} E(X, j)y_j.$$  \hfill (9)

III. THEORETICAL FRAMEWORK FOR PROPOSED METHOD

The core of our proposed method to find the best strategy and a way to simplify its implementation are presented in the followings as theorems. The first theorem allows us to use the majorization theory [2] to find the value of the game for the player $P_1$.

**Theorem 1:** Let $E$ be the expectation function of an $m \times n$ rectangular game, let $v$ be a real number, and let $X^*$ and $Y^*$ be members of $S_m$ and $S_n$, respectively. Then a necessary and sufficient condition that $v$ be the value of the game and that $X^*$ and $Y^*$ be optimal strategies for $P_1$ and $P_2$, respectively, is that for $1 \leq i \leq m$ and $1 \leq j \leq n$:

$$E(i, Y^*) \leq v \leq E(X^*, j).$$ \hfill (10)

**Proof:** By replacing $X$ by $X_i$ and $Y$ by $Y_j$ the necessity follows directly from the definition of the value of the game given in (8). Assuming that (9) is true then $\forall X \in S_m$ we have

$$\sum_{i=1}^{n} E(i, Y^*)x_i \leq \sum_{i=1}^{n} v x_i = v$$ and thus:

$$E(X, Y^*) \leq v.$$ \hfill (11)

Similarly, we obtain $\forall Y \in S_n$:

$$v \leq E(X^*, Y).$$ \hfill (12)

From (24) and (25) by replacing $X$ by $X^*$ and $Y$ by $Y^*$ we obtain:

$$E(X^*, Y^*) \leq v \leq E(X^*, Y^*)$$ \hfill (13)

and, according to (8), $v$ is the value of game and the pair $[X^*, Y^*]$ is indeed a saddle point. $\Box$.

In the particular case $X = Y = p^*$ and $m = n = K$, by using (5), the value of the game can be expressed in terms of the eigenvalues of the weighted Gram matrix $W = P^{12}G P^{12}$. For the eigenvalues of this matrix, the following majorization relation stands ($K > N$) [2]:

$$[0, ..., 0, \sum_{i=1}^{K} \lambda_i] \prec \lambda_1, \ldots, \lambda_K, 0, \ldots, 0] \prec [0, ..., 0, 0, ..., 0].$$ \hfill (14)

Using the fact that for any Hermitian matrix the trace is the sum of its eigenvalues $\sum_{i=1}^{K} \lambda_i = \sum_{i=1}^{K} p_i = p_{tot}$ [4] we have:

$$[0, ..., 0, p_{tot}] \prec [\lambda_1, \ldots, \lambda_K, 0, \ldots, 0] \prec [p_{tot}, 0, 0, \ldots, 0].$$ \hfill (15)

It easy to check that:

$$[0, ..., 0, p_{tot}] \prec \left(\frac{p_{tot}}{N}\right)^2 \prec [p_{tot}, 0, 0, \ldots, 0].$$ \hfill (16)

Using (16) and (15) in (5) and based on (7) we obtain the value of the game $v$ satisfying:

$$\left(\frac{p_{tot}}{N}\right)^2 / N \leq v \leq \left(\frac{p_{tot}}{N}\right)^2 / N.$$ \hfill (17)

By Theorem 1 the value of this game is $v = \left(\frac{p_{tot}}{N}\right)^2 / N$, which corresponds to the minimum TWSC [7], [15]. The following theorem assures that the above value of the game is a strategic saddle point for both players and will allows us to find the optimal strategies for both of the players.

**Theorem 2:** Let $E$ be the expectation function of a $m \times n$ rectangular game whose value is $v$. Then a necessary and sufficient condition that $X^* \in S_m$ be an optimal strategy for the player $P_1$ is that for every $Y \in S_n$ we have $v \leq E(X^*, Y)$. Similarly, a necessary and sufficient condition that $Y^* \in S_n$ be an optimal strategy for the player $P_2$ is that for $X \in S_m$ we have $E(X, Y^*) \leq v$.

**Proof:** Let us assume that there exist a point $[X^*, Y^*]$ such that $\forall X \in S_m$ and $\forall Y \in S_n$ we have, according to (8):

$$E(X, Y^*) \leq E(X^*, Y^*) \leq E(X', Y')$$ \hfill (18)
and since, by hypothesis, \( v \) is the value of the game we have \( E(\mathbf{s}, \mathbf{y}) = v \). Now suppose that \( \mathbf{x} \in S_\pi \) such that \( \forall \mathbf{y} \in S_\pi \), we have \( v \leq E(\mathbf{x}', \mathbf{y}) \). So we can conclude that \( E(\mathbf{x}, \mathbf{y}) \leq E(\mathbf{x}', \mathbf{y}) \). Replacing \( \mathbf{x} \) by \( \mathbf{x}' \) in the first part of (18) we obtain:

\[
E(\mathbf{x}', \mathbf{y}) \leq E(\mathbf{x}, \mathbf{y}) \leq E(\mathbf{x}', \mathbf{y})
\]  

(19)

and thus \( E(\mathbf{x}, \mathbf{y}) \leq E(\mathbf{x}', \mathbf{y}) \). From (18), (19) we now conclude that:

\[
E(\mathbf{x}', \mathbf{y}) \leq E(\mathbf{x}', \mathbf{y}) \leq E(\mathbf{x}, \mathbf{y})
\]  

(20)

so that the point \( [\mathbf{x}', \mathbf{y}] \) is a saddle point of \( E \), and hence \( \mathbf{x}' \) is an optimal strategy for the player \( P_1 \). The proof of the second part of the theorem is similar. □

The above theorem simply tells us that it is enough to derive the optimal strategy for the player \( P_1 \).

IV. CONCLUSIONS

In this paper by using a game theoretical approach we defined the Total Weighted Squared Correlation as an extensive game payoff and we derived a method to obtain the optimal strategy for this game. By using the majorization theory we derived the saddle point and this coincide with the minimum TWSC.

A practical algorithm implementing the proposed method, numerical results and comparisons with previous methods are the subject of a companion paper [31].

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