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Implementation of an OFDM Synchronizer

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Abstract – The development of faster signal processing components and technologies recently induced an increased interest in Multicarrier or Orthogonal Frequency Division Multiplexing (OFDM); this is a multiplexing technique which converts a frequency-selective fading channel into several nearly flat-fading channels and combats the intersymbol interferences (ISI) caused by multipath propagation. One of the problems of the OFDM technique is the synchronization of the receiver, so this paper will focus on how synchronization can be achieved and implemented.

Keywords: OFDM, synchronization, timing offset, carrier frequency offset.

I. INTRODUCTION

OFDM is used in some wireless communications applications, like WLAN, being included in standards as IEEE 802.11 (USA), ARIB MMAC (Japan), HIPERLAN/2 (ETSI BRAN Europe). Also OFDM is employed for Digital Audio Broadcast (DAB) applications and known as Digital Multitone (DMT) for broadband wireline communication systems, namely high-bit-rate / asymmetric digital subscribers line (HDSL / ADSL) [1]. Advantages of OFDM are that it is bandwidth efficient and that it is rather insensitive to frequency selective fading and timing offset. The most important disadvantage though is that OFDM is sensitive to carrier frequency offset (CFO).

II. SYSTEM MODEL

An OFDM symbol can be constructed as suggested in Fig. 1 [2], [6]. First, the data to be transmitted is mapped to a complex value X_k in the frequency domain, according to a QAM signal constellation. Second, the IDFT is calculated, normally using an IFFT algorithm, to get a complex time domain OFDM symbol

$$x_n = IFFT(X_k) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{-j\frac{2\pi kn}{N}} \qquad (1)$$

where N is the number of subcarriers. To make OFDM more robust against multipath and timing offset, each symbol is extended with a guard interval or a cyclic prefix (CP). The CP is constructed by copying the last N_g samples of the OFDM symbol (T_{es} being the sampling period) at the beginning of it. Thus, the OFDM symbol transmitted is $x_{-N_g}\, x_{-N_g+1} \ldots\ldots x_{N-2}\, x_{N-1}$. Finally, the time domain signals are D-A converted, mixed with a carrier, filtered and transmitted through the air. In the receiver, the opposite operations are performed using A-D conversion and DFT calculation. Since QAM uses coherent detection, the receiver has to estimate the phase to be able to successfully recover the information that was sent. Coherent detection means that all the signal alternatives are known in the receiver. It also has to compensate for different distorsion caused by the channel.

In order to give a mathematical description of an OFDM system we assume a system with N subcarriers, a bandwidth of B Hz and an OFDM symbol length of T_S seconds, of which T_{CP} is the length of the cyclic prefix. The spacing between subcarriers is given by (2).

$$T = \frac{1}{\Delta f} = \frac{N}{B} = T_S - T_{CP} \qquad (2)$$

Fig. 2 illustrates the baseband OFDM model mathematically described bellow [1], [3], [4]. Every n^{th} OFDM symbol of the transmission stream can be written as a set of modulated carriers transmitted in parallel. Relations (3) express the waveforms used in modulation.

$$\phi_k(t) = \begin{cases} \frac{1}{\sqrt{T_S - T_{CP}}} \cdot e^{j2\pi f_k (t - T_{CP})} & , t \in [0, T_S) \\ 0 & , \text{otherwise} \end{cases}, \text{ where}$$

$$f_k = f_C + \left(k - \frac{N-1}{2}\right) \cdot \frac{1}{T}, \quad k = 0, \ldots, N-1, \text{ for passband or} \qquad (3)$$

$$f_k = \frac{k}{T}, \quad k = 0, \ldots, N-1, \text{ for baseband echivalent}$$

Note that nonzero term of $\phi_k(t)$ has the period $[T_{CP}, T_S)$ and $\phi_k(t)$ has a common part (4).

$$\phi_k(t) = \phi_k\left(t + \frac{N}{B}\right), \quad \text{for } t \in [0, T_{CP}) \qquad (4)$$

If $d_{n,0}, \ldots, d_{n,N-1}$ denotes the complex symbols, obtained by QAM mapping of the input data stream, the n^{th} OFDM symbol $s_n(t)$ is expressed by (5) and the infinite sequence of OFDM symbols transmitted is obtained by juxtaposition of the individual ones.

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$$s(t)=\sum_{n=-\infty}^{\infty}s_n(t)=\sum_{n=-\infty}^{\infty}\sum_{k=0}^{N-1}d_{k,n}\phi_k(t-nT_S) \quad (5)$$

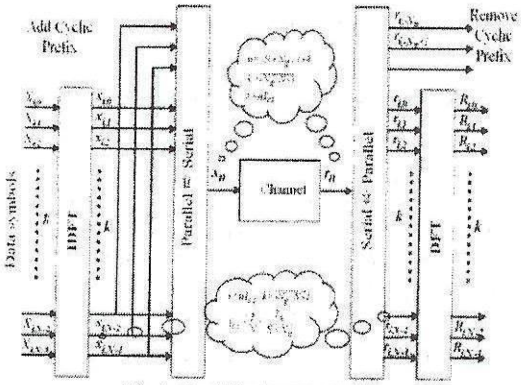

Fig. 1. The OFDM system model.

Assuming the impulse response $ch(\tau;t)$ of the physical channel (possibly time variant) is restricted to the length of cyclic prefix $\tau\in[0,T_{CP})$, the received signal becomes (6), where $n(t)$ is the complex, additive and white Gaussian (AWGN) channel noise.

$$r(t)=(ch*s)(t)=\int_0^{T_{CP}}ch(\tau;t)s(t-\tau)+n(t) \quad (6)$$

The filter from the receiver is matched to the last part $[T_{CP},T_S)$ of the transmitter waveform (7), the CP being this way effectively removed in the receiver. Since the cyclic prefix contains the ISI, the sample output from the receiver filter bank contains no ISI. Also, we can ignore the time index n when calculating the sampled output at the k^{th} matched filter (8).

$$\psi_k(t)=\begin{cases}\phi_k^*(T_S-t) & ,t\in[0,T_S-T_{CP})\\ 0 & ,\textit{otherwise}\end{cases} \quad (7)$$

$$e_k=(r*\psi_k)(t)\big|_{t=T_S}=\int_{-\infty}^{\infty}r(t)\cdot\psi_k(T_S-t)\cdot dt \quad (8)$$

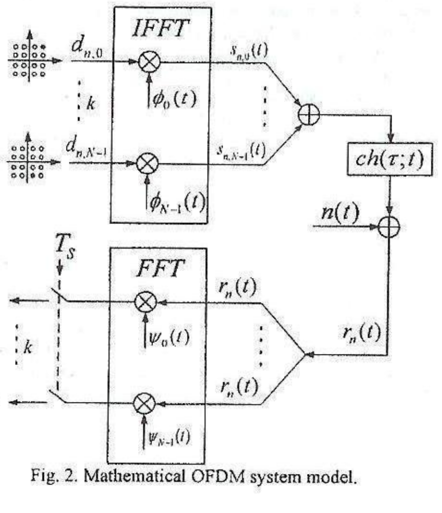

Fig. 2. Mathematical OFDM system model.

Considering the channel to be fixed over the OFDM symbol interval, denoting it by $ch(\tau)$ and taking into account the orthogonality condition expressed by (9), we obtain after some mathematical operations the output data, given by (10).

$$\int_{T_{CP}}^{T}\phi_l(t)\cdot\phi_k^*(t)=\delta(k-l) \quad (9)$$

$$e_k=h_k\cdot d_k+n_k\,,\quad \textit{where}$$

$$h_k=\int_0^{T_{CP}}ch(\tau)\cdot e^{-j2\pi k\tau\frac{B}{N}}\cdot d\tau \;\textit{and} \quad (10)$$

$$n_k=\int_{T_{CP}}^{T_S}n(T_S-t)\cdot\phi_k^*(t)\cdot dt$$

By sampling the low-pass equivalent signal of (3) and (5) at a rate N times higher than the subcarrier symbol rate $1/T$, we can obtain the discrete model of the baseband OFDM system, where the modulation/demodulation with waves ϕ/ψ can be replaced with iDFT/DFT (or practically with IFFT/FFT) and the channel model with discrete-time convolution, like in Fig. 1.

III. SYNCHRONIZATION PROBLEM

It is important to know what happens when the received signal is not synchronized in time and frequency and how to counteract this effects. The synchronization must address a few issues. First, the receiver has to estimate the symbol boundaries and the optimal timing instants that minimize the effects of inter-carrier and inter-symbol interferences. Second, the receiver has to estimate and correct for the carrier frequency offset of the received signal, because the subcarriers are orthogonal only if the receiver and the transmitter use the same frequencies. Further, the phase information must be recovered if coherent demodulation is employed [8], [9], [13].

The most challenging is the timing and frequency synchronization, which is considered in this paper, neglecting the sampling frequency offset and the phase tracking. The timing offset is the difference between the estimated timing instant and the correct timing instant. The timing offset τ_0 smaller than the CP length can be seen as if each complex OFDM symbol is multiplied by $e^{j\frac{2k\pi\tau_0}{NT_{es}}}$, where k is the subchannel number. The multiplication causes a subchannel dependent rotation in the complex plane. As long as the timing offset is small enough this rotation can be estimated and corrected in the channel estimator or in the phase tracker. Because of the differences between the oscillators in the transmitter and receiver, Doppler shift, etc. each path in the received signal is affected by a carrier frequency offset (CFO) Δf, which has the same effect as if each sample n is multiplied by $e^{j2\pi n\Delta fT_{es}}$, causing a time varying rotation in the complex plane. Since a coherent system has to calculate and compensate the phase continuously, for example by using the known pilot subcarriers, the rotation can be estimated and compensated for [11], [12], [14].

IV. SYNCHRONIZER IMPLEMENTATION

In order to estimate the CFO, in this paper a time domain maximum likelihood (ML) estimation is considered, using a preamble with short and long training symbols as in the IEEE802.11 standard, where, to help the receiver accomplish synchronized reception, a known data sequence, the preamble, is transmitted at the beginning of each packet. The first half of the preamble consists of ten identical short symbols. Each short symbol consists of 16 samples. The second half of the preamble consists of two identical long symbols, each 64 samples long, preceded by a 32-sample CP. The symbols are designed so that the correlation between two subsequent samples is minimal.

To estimate the CFO, first the correlation between the first and the last N_g samples has to be calculated, where N_g is the number of the samples in the CP.

$$R=\sum_{k=0}^{N_g-1} r(k)r^*(k+N) \tag{11}$$

The estimate of the CFO is found by finding the angle of this correlation.

$$\hat{C}FO=-\frac{1}{2\pi}\cdot\arg R \tag{12}$$

The preamble is designed to aid the CFO estimation. Each short training symbol can be seen as a CP to the other short symbols, making it easy to calculate an average. The same is the case for the long training symbols.

Even after the CFO has been corrected, in reality, there will always exist a small residual frequency offset (RFO). This RFO can be used to track the CFO by measuring the phase rotation between two OFDM symbols. The most natural way to compensate for CFO would be to simply feed back the estimated CFO to the oscillator in the receiver. The compensation can, however, be done digitally by multiplying each incoming sample by $e^{-j2\pi n\Delta f T_{es}}$.

Also, by comparing continuously the correlation for the short training symbols and the received energy and taking into account that they should be the same when the received symbol contains the short training symbols, the timing estimation can be accomplish. This method gives a packet detection solution, but a further fine timing estimation can later be found by correlating the incoming symbols with the known long symbols and then try to find the maximizing instant [7].

In Fig. 3 the structure of CFO estimator and corrector is presented. At the start of the packet, the incoming samples pass through rotator unchanged and into the correlator [7]. The output from the correlator is then transferred into the angle calculator, which computes the argument of the complex signal. The mean is then calculated and finally a CFO estimate is found. The first estimate is then used to decrease the CFO before the new estimate is calculated using long training symbols.

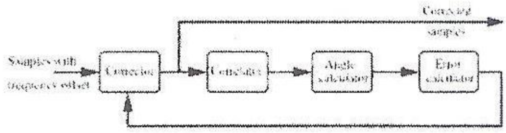

Fig. 3. The structure of the frequency estimator and corrector.

V. CONCLUSIONS

The method presented for timing and frequency synchronization is based on measuring the phase difference between the cyclic prefix and the symbol during the short and long training symbols in the packet preample. The OFDM system has been implemented using Matlab and Simulink [5], [10], while the CFO estimator and corrector was implemented and integrated in the main Matlab model as a HDL synthesizable model [6].

The simulations show acceptable performance in an AWGN (Additive White Gaussian Noise) channel. At an SNR of 25 dB the mean remaining CFO after the correction was 0.2% of the distance between two subcarriers. In a multipath environment expected performance degradation was observed.

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